

## Problem Set #2: Expected Value and Insurance

1. An individual with initial wealth of \$400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose \$300, leaving him with \$100; if he doesn't, he loses nothing. He maximizes expected utility, and his VNM utility function is  $u(w) = \sqrt{w}$ .
  - (a) What is the  $u''(w)$ , i.e., the second derivative of this person's utility function?
  - (b) What is the expected amount of money he will lose? What is his expected wealth?
  - (c) What is his expected utility?
  - (d) What is his **certainty equivalent wealth**, i.e., the certain wealth level that gives him the same expected utility as his uncertain situation?
  - (e) What is the maximum amount he would pay for **full insurance**, i.e., what is the maximum premium he would pay an insurance company to cover all of his losses?
  - (f) What is his **risk premium**, i.e., how much more than his expected loss is he willing to pay for full insurance?
  - (g) How do your answers to the above questions change if his utility function is  $u(w) = 3\sqrt{w} + 20$  instead of  $u(w) = \sqrt{w}$ . Explain.
2. An individual with initial wealth of \$400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose \$300, leaving him with \$100; if he doesn't, he loses nothing. He maximizes expected utility, and his VNM utility function is  $u(w) = 20w + 10$ .
  - (a) What is the  $u''(w)$ , i.e., the second derivative of this person's utility function?
  - (b) What is the expected amount of money he will lose? What is his expected wealth?
  - (c) What is his expected utility?
  - (d) What is his certainty equivalent wealth?
  - (e) What is the maximum amount he would pay for full insurance?
  - (f) What is his risk premium, i.e., how much more than his expected loss is he willing to pay for full insurance?
3. An individual with initial wealth of \$400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose \$300, leaving him with \$100; if he doesn't, he loses nothing. He maximizes expected utility, and his VNM utility function is  $u(w) = w^2$ .

- (a) What is the  $u''(w)$ , i.e., the second derivative of this person's utility function?
  - (b) What is the expected amount of money he will lose? What is his expected wealth?
  - (c) What is his expected utility?
  - (d) What is his certainty equivalent wealth?
  - (e) What is the maximum amount he would pay for full insurance?
  - (f) What is his risk premium, i.e., how much more than his expected loss is he willing to pay for full insurance?
4. Consider an expected-utility-maximizing individual with VNM utility function  $u(w) = \sqrt{w}$  and initial wealth \$400. With probability  $p$  she will get into an accident that will result in a loss of \$300; with probability  $(1 - p)$  she loses nothing. Imagine that she can buy insurance, but that the insurance company charges her an \$8 application fee in addition to her expected loss.
- (a) Draw a graph with  $w_1$ , her wealth if she doesn't get into an accident, on the  $x$ -axis, and  $w_2$ , her wealth if she does get into an accident, on the  $y$ -axis. Then represent the following on the graph (not with mathematical precision, just with the ideas): her initial endowment point, indicating its coordinates; the indifference curve containing her endowment point; the fair-odds line for actuarially fair insurance (i.e., the set of all points that an actuarially fair policy would cover), indicating the equation for this line; and the fair-odds line for the insurance with the \$8 application fee (i.e., the set of all points that could be covered with an \$8 fee plus whatever her expected loss is), indicating the equation for this line. *Indicate on the graph what distance(s), if any, represent(s) the \$8 application fee.*
  - (b) For what values of  $p$  will she purchase insurance? You should find that she will choose to **self-insure** for values of  $p$  close to either 0 or 1; *provide an intuitive explanation for this result.*
  - (c) For a random variable  $x$ , the **variance** of  $x$ ,  $\text{Var}(x)$ , is defined as

$$\text{Var}(x) = E [(x - E(x))^2].$$

Calculate the variance of this individual's loss and the variance of her wealth (when you do this, you should assume that she does not have insurance); then compare the two; then note any apparent connections between these variances and her choice about whether or not to buy insurance for different values of  $p$ .

- (d) How do your answers to the above questions change if her utility function is  $u(w) = 3\sqrt{w} + 20$  instead of  $u(w) = \sqrt{w}$ . Explain.