Problem Set #3: Consumer Theory

1. Consider an individual with a utility function of $U(L, K) = L^2K$ and a budget of M.
   (a) Write down the utility-maximization problem. Clearly specify the objective function and the choice variables. Use p_L and p_K as the prices of L and K.
   (b) Explain how to go about solving this problem if you were given values of M, p_L, and p_K.
   (c) Assume the prices of L and K are $p_L = 2$ and $p_K = 3$, and that the individual's budget is $M = 90$. Find the maximum level of utility this individual can reach with a budget of 90. What are the utility-maximizing choices of L and K?
   (d) Calculate the individual's Marshallian demand curve for lattes[1] if the price of K is $p_K = 3$ and the budget is $M = 90$. Use p_L for the price of lattes. What is the slope of this demand curve when $p_L = 2$?
   (e) Calculate the individual's Marshallian demand curve for cake if the price of L is $p_L = 2$ and the budget is $M = 90$. Use p_K for the price of cake. What is the slope of this demand curve when $p_K = 3$?
   (f) Calculate the individual's Engel curves for lattes and cake[2] if the price of lattes is $p_L = 2$ and the price of cake is $p_K = 3$. Use M for the individual's budget.

2. Consider the same individual as above, with utility function $U(L, K) = L^2K$. This time assume that the individual has a utility constraint of $U = 9000$.
   (a) Write down the cost-minimization problem. Clearly specify the objective function and the choice variables. Use p_L and p_K as the prices of L and K.
   (b) Explain how to go about solving this problem if you were given values of p_L, and p_K.
   (c) Assume the prices of L and K are $p_L = 2$ and $p_K = 3$. Find the minimum budget required to reach a utility level of 9000. What are the cost-minimizing choices of L and K?
   (d) Calculate the individual's Hicksian demand curve for lattes[3] if the price of K is $p_K = 3$. Use p_L for the price of lattes. Calculate the slope of this demand curve when $p_L = 2$.

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[1]The **Marshallian demand curve** for lattes shows the relationship between the price of lattes and the quantity of lattes demanded, holding all other prices *and the individual's budget* constant. Marshallian demand curves are also called "money-held constant" demand curves, and the repeated "m" makes for a useful pneumonic.

[2]The **Engel curve** for lattes shows the relationship between the individual's *income* and the quantity of lattes demanded, holding all prices constant.

[3]The **Hicksian demand curve** for lattes shows the relationship between the price of lattes and the quantity of lattes demanded, holding all other prices *and the individual's utility level* constant. Hicksian demand curves are also called "utility-held constant" demand curves.

(e) Calculate the individual's Hicksian demand curve for cake if the price of L is $p_L = 2$. Use p_K for the price of cake. Calculate the slope of this demand curve when $p_K = 3$.

3. The previous problem (about Hicksian demand curves) asked you to minimize cost subject to a utility constraint of $U = 9000$; you should have gotten a minimum budget of $M = 90$. The problem before that (about Marshallian demand curves) asked you to maximize utility subject to a budget constraint of $M = 90$; you should have gotten a maximum utility of $U = 9000$. Explain why these two problems give symmetric answers.

4. This question examines the relationship between "willingness to pay" and the area under demand curves.

   (a) Consider (as above) an individual with utility $U(L, K) = L^2 K$ who is facing prices of $p_L = 2$ and $p_K = 3$. We know from above that this individual can reach a utility level of 9000 at a minimum cost of \$90 by choosing $L = 30$, $K = 10$. Write down a *formal question* that corresponds to this informal question: what is this individual's willingness-to-pay for 10 more lattes?

   (b) Solve this problem by answering the following questions: (1) If we give this individual 30 lattes, how much money do they have to spend on cake to reach a utility of 9000? (2) If we give this individual 40 lattes, how much money do they have to spend on cake to reach a utility of 9000? (3) Subtract. [Note: You can also do this problem by assuming that this individual *buys* the first 30 lattes at a price of \$2 each. The answers to (1) and (2) will be \$60 higher, but the answer to (3) will be the same.]

   (c) Recall from question 2d above that the Hicksian demand curve for lattes is a rearrangement of this equation: $L^3 = 54000 p_L^{-1}$. Instead of rearranging this to get the demand curve $L = \left(54000 p_L^{-1}\right)^{\frac{1}{3}} = (30)2^{\frac{1}{3}} p_L^{-\frac{1}{3}}$, rearrange it to get an **inverse Hicksian demand curve**, i.e., an expression for p_L as a function of L

   (d) Draw a graph of this inverse demand curve. Shade in the area between $L = 30$ and $L = 40$.

   (e) Calculate the area under the inverse Hicksian demand curve between $L = 30$ and $L = 40$. Compare with your willingness-to-pay calculation above.

   (f) Can you think of a *formal question* that is answered by the area under a *Marshallian* demand curve?