Answer Key to PS #3: Consumer Theory

1. (a) The individual wants to choose L and K to maximize utility $U(L,K) = L^2K$ subject to the budget constraint $p_L L + p_K K = M$.

   (b) The solution method is to find two equations involving L and K and then solve them simultaneously. One equation is the constraint, $p_L L + p_K K = M$; the other is the last dollar rule, $\frac{\frac{\partial U}{\partial L}}{p_L} = \frac{\frac{\partial U}{\partial K}}{p_K}$.

   (c) The last dollar rule gives us $\frac{2LK}{p_L} = \frac{L^2}{p_K}$, which simplifies to $3K = L$ when $p_L = 2$ and $p_K = 3$. Substituting this into the budget constraint we have $2(3K) + 3K = 90$, i.e., $K = 10$. Substituting back into either of our equations yields $L = 30$.

   (d) The last dollar rule gives us $\frac{2LK}{p_L} = \frac{L^2}{p_K}$, which simplifies to $6K = p_L L$ when $p_K = 3$. Solving for K and substituting this into the budget constraint yields $p_L L + 3\left(\frac{1}{6}p_L L\right) = 90$, i.e., $1.5 p_L L = 90$. This simplifies to $L = \frac{60}{p_L}$, which is the Marshallian demand curve for lattes when $p_K = 3$ and $M = 90$. The slope of this demand curve is $\frac{dL}{dp_L} = -60p_L^{-2}$; when $p_L = 2$, this simplifies to $\frac{-60}{4} = -15$.

   (e) The last dollar rule gives us $\frac{2LK}{p_L} = \frac{L^2}{p_K}$, which simplifies to $p_K K = L$ when $p_L = 2$. Using this to substitute for L in the budget constraint yields $2(p_K K) + p_K K = 90$, i.e., $K = \frac{30}{p_K}$, which is the Marshallian demand curve for cake when $p_L = 2$ and $M = 90$. The slope of this demand curve is $\frac{dK}{dp_K} = -30(p_K)^{-2}$; when $p_K = 3$, this simplifies to $-\frac{30}{9} \approx -3.33$.

   (f) The last dollar rule gives us $\frac{2LK}{p_L} = \frac{L^2}{p_K}$, which simplifies to $3K = L$ when $p_L = 2$ and $p_K = 3$. Using this to substitute for L in the budget constraint yields $2(3K) + 3K = M$, i.e., $K = \frac{M}{9}$, which is the Engel curve for cake when $p_L = 2$ and $p_K = 3$. Using the last dollar rule result $3K = L$ to substitute for K in the budget constraint yields $2L + L = M$, i.e., $L = \frac{M}{3}$, which is the Engel curve for cake when $p_L = 2$ and $p_K = 3$.

2. (a) Choose L and K to minimize $C(L,K) = p_L L + p_K K$ subject to the constraint $L^2K = 9000$.

   (b) Combine the constraint with the last dollar rule to get two equations in two unknowns. Solve these simultaneously to get the optimal values of L and K.

   (c) We previously calculated the last dollar rule to yield $3K = L$ when $p_L = 2$ and $p_K = 3$. Substituting this into the constraint yields $(3K)^2K = 9000$, which simplifies to $9K^3 = 9000$, i.e., $K = 10$. It follows from either equation that $L = 30$. The cost-minimizing cost is therefore $p_L L + p_K K = 2(30) + 3(10) = 90$.

(d) The last dollar rule gives us $\frac{2LK}{p_L} = \frac{L^2}{p_K}$, which simplifies to $6K = p_L L$ when $p_K = 3$. Using this to substitute for K in the constraint $L^2K = 9000$ yields $L^2 \frac{p_L L}{6} = 9000$, which simplifies to $L^3 = 54000 p_L^{-1}$, and then to $L = \left(54000 p_L^{-1}\right)^{\frac{1}{3}} = (30)2^{\frac{1}{3}} p_L^{-\frac{1}{3}}$. The slope of this demand curve is $\frac{dL}{dp_L} = -\frac{1}{3}(30)2^{\frac{1}{3}} p_L^{-\frac{4}{3}}$. When $p_L = 2$ this simplifies to $(-10)2^{-1} = -5$.

(e) The last dollar rule gives us $\frac{2LK}{p_L} = \frac{L^2}{p_K}$, which simplifies to $p_K K = L$ when $p_L = 2$. Using this to substitute for L in the constraint $L^2K = 9000$ yields $(p_K K)^2 K = 9000$, which simplifies to $p_K^2 K^3 = 9000$, and then to $K = \left(9000 p_K^{-2}\right)^{\frac{1}{3}} = (10)3^{\frac{2}{3}} p_K^{-\frac{2}{3}}$. The slope of this demand curve is $\frac{dK}{dp_K} = -\frac{2}{3}(10)3^{\frac{2}{3}} p_k^{-\frac{5}{3}}$. When $p_K = 3$ this simplifies to $-\frac{2}{3}(10)3^{-1} = -\frac{20}{9} \approx -2.22$.

3. The two problems are two sides of the same coin: if $U = 9000$ is the maximum utility that can be achieved with a budget of $M = 90$, then $M = 90$ is the minimum budget required to reach a utility level of $U = 9000$.

4. (a) What is the maximum amount of money this individual could exchange for 10 more lattes *and still be on the same indifference curve*, i.e., still have a utility level of 9000?

(b) We must have $L^2K = 9000$. So (1) if $L = 30$ then we need $K = 10$, which at a price of \$3 per cake requires a budget of \$30; (2) if $L = 40$ then we need $K = 5.625$, which at a price of \$3 per cake requires a budget of \$16.875; (3) subtracting yields \$30-\$16.875 = \$13.125 as this individual's willingness-to-pay for those 10 lattes.

(c) $p_L = 54000 L^{-3}$

(d) See figure 1.

(e) The area is the same as the willingness-to-pay calculated above!

$$\int_{30}^{40} 54000 L^{-3} dL = -27000 L^{-2} \Big|_{30}^{40} = -16.875 + 30 = \$13.125$$

(f) Various texts (e.g., Silberberg's *Structure of Economics* insist that the area under Marshallian demand curves is meaningless.

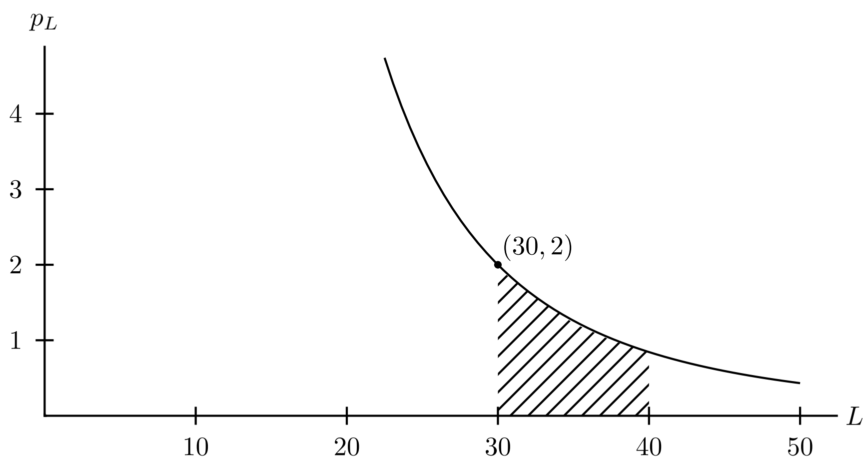

Figure 1: The area under the *Hicksian* (utility-held-constant) demand curve measures willingness to pay.