Answer Key for Part I: The Optimizing Individual

Chapter 1: Decision Theory

1. A newspaper column in the summer of 2000 complained about the overwhelming number of hours being devoted to the Olympics by NBC and its affiliated cable channels. The columnist argued that NBC had established such an extensive programming schedule in order to recoup the millions of dollars it had paid for the rights to televise the games. Do you believe this argument? Why or why not?

   You should not believe this argument because the amount spent on the rights to the Olympic Games is a sunk cost. The reason the network showed the Olympics for so many hours was because that was the decision that maximized profits.

2. You win \$1 million in the lottery, and the lottery officials offer you the following bet: You flip a coin; if it comes up heads, you win an additional \$10 million. If it comes up tails, you lose the \$1 million. Will the amount of money you had prior to the lottery affect your decision? (*Hint:* What would you do if you were already a billionaire? What if you were penniless?) What does this say about the importance of sunk costs?

   A billionaire would probably take this bet because its expected value is \$4.5 million. A penniless person would probably not take the bet, because the risk of ending up with zero is too great. So sunk costs cannot be entirely ignored in decision-making; rather, the point is that it is not possible to base decisions *solely* on sunk costs.

3. Alice the axe murderer is on the FBI's Ten Most Wanted list for killing six people. If she is caught, she will be convicted of these murders. The state legislature decides to get tough on crime, and passes a new law saying that anybody convicted of murder will get the death penalty. Does this serve as a deterrent for Alice, i.e., does the law give Alice an incentive to stop killing people? Does the law serve as a deterrent for Betty, who is thinking about becoming an axe murderer but hasn't killed anybody yet?

The law does not deter Alice from committing additional crimes because she's already facing the death penalty if she's caught. The law does deter Betty, because she hasn't killed anybody yet.

4. A pharmaceutical company comes out with a new pill that prevents baldness. When asked why the drug costs so much, the company spokesman replies that the company needs to recoup the $10 billion it spent on research and development.

   (a) Do you believe the spokesman's explanation?

   (b) If you said "Yes" above: Do you think the company would have charged less for the drug if it had discovered it after spending only $5 million instead of $10 billion?

   If you said "No" above: What alternative explanation might help explain why the drug company charges so much for its pill?

   You should not believe the spokesman's explanation because the R&D expenditure is a sunk cost. If it spent twice as much or half as much to discover the drug, it should still charge the same price, because that's the price that maximizes profit. And that's the alternative explanation for why the drug company charges such a high price: that's the price that maximizes profit.

Chapter 2: Optimization and Risk

1. You roll a six-sided die and win that amount (minimum $1, maximum $6). What is the expected value of this game?

   The expected value is $\frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6) = \frac{21}{6}$.

2. With probability 1/3 you win $99, with probability 2/3 you lose $33. What is the expected value of this game?

   The expected value is $\frac{1}{3}(99) + \frac{2}{3}(-33) = 11$.

3. *Fun/Challenge* (The Monty Hall Problem) The Monty Hall Problem gets its name from the TV game show *Let's Make A Deal*, hosted by Monty Hall. The scenario is this: Monty shows you three closed doors. Behind one of these doors is a new car. Behind the other two doors are goats (or some other "non-prize"). Monty asks you to choose a door, but after you do he *does not* show you what is behind the door you chose. Instead, he opens one of the *other* doors, revealing a goat, and then offers you the opportunity to switch to the remaining unopened door. [As an example, say you originally pick Door #1. Monty opens up Door #2, revealing a goat, and then offers you the opportunity to switch to from Door #1 to Door #3.] What should you do?

   You should switch: your odds of winning will increase from $\frac{1}{3}$ to $\frac{2}{3}$. A more extreme example may help provide some intuition behind this result:

assume that there are 100 doors, only one of which leads to a car; after you pick a door, Monty opens up 98 of the other doors to reveal goats and then offers you the opportunity to switch to the remaining unopened door. Doing so will increase your odds of winning from $\frac{1}{100}$ to $\frac{99}{100}$.

Still confused? Go online[1] for a fun computer simulation of the Monty Hall problem (with accompanying discussion).

4. Imagine that you are taking a multiple-guess exam. There are five choices for each question; a correct answer is worth 1 point, and an incorrect answer is worth 0 points. You are on Problem #23, and it just so happens that the question and possible answers for Problem #23 are in Hungarian. (When you ask your teacher, she claims that the class learned Hungarian on Tuesday. . . .)

   (a) You missed class on Tuesday, so you don't understand any Hungarian. What is the expected value of guessing randomly on this problem?

   The expected value of guessing randomly is $\frac{1}{5}(1) + \frac{4}{5}(0) = \frac{1}{5}$.

   (b) Now imagine that your teacher wants to discourage random guessing by people like you. To do this, she changes the scoring system, so that a blank answer is worth 0 points and an incorrect answer is worth x, e.g., $x = -\frac{1}{2}$. What should x be in order to make random guessing among five answers a fair bet (i.e., one with an expected value of 0)?

   If an incorrect answer is worth x, the expected value from guessing randomly is $\frac{1}{5}(1) + \frac{4}{5}(x) = \frac{1+4x}{5}$. If the teacher wants this expected value to equal zero, she must set $x = -\frac{1}{4}$.

   (c) Is the policy you came up with in the previous part going to discourage test-takers who are risk-averse? What about those who are risk-loving?

   Since this makes random guessing a fair bet, it will discourage risk averse students but not risk loving students.

   (d) Your teacher ends up choosing $x = -\frac{1}{3}$, i.e., penalizing people 1/3rd of a point for marking an incorrect answer. How much Hungarian will you need to remember from your childhood in order to make guessing a better-than-fair bet? In other words, how many answers will you need to eliminate so that guessing among the remaining answers yields an expected value strictly greater than 0?

   If you can't eliminate any answers, the expected value of guessing randomly is $\frac{1}{5}(1) + \frac{4}{5}\left(-\frac{1}{3}\right) = -\frac{1}{15}$. If you can eliminate one answer, you have a 1 in 4 chance of getting the right answer if you guess randomly, so your expected value if you can eliminate one answer is $\frac{1}{4}(1) + \frac{3}{4}\left(-\frac{1}{3}\right) = 0$. If you can eliminate two answers, you have a 1 in 3 chance of getting the right answer if you guess randomly, so your

[1] http://cartalk.cars.com/About/Monty/

expected value if you can eliminate two answers is $\frac{1}{3}(1) + \frac{2}{3}\left(-\frac{1}{3}\right) = \frac{1}{9}$. So you need to eliminate at least two answers in order to make random guessing yield an expected value greater than zero.

5. Two businesses that involve lots of gambling are the casino business and the insurance business. Are these businesses particularly risky to get involved in? Explain why or why not.

   No, they are not particularly risky. This is because of the **law of large numbers**, discussed in Section 2.1. The individual bettor plays roulette only a few times, and so faces a lot of risk. The casino plays roulette thousands of times each day, and so has a very good idea of what the overall outcome will be; since each \$1 wager has an expected payoff of only \$.95, it can expect to gain about \$.05 for every dollar wagered.

   Similarly, although insurance companies have no idea whether an individual driver is going to get into an accident this year, or whether an individual person is going to die this year, or whether an individual home is going to burn to the ground this year, the law of large numbers usually gives them a very good idea of the percentage of accidents or the percentage of deaths or the percentage of fires to expect from the hundreds of thousands of cars, lives, and homes they cover.

   So casinos or insurance companies are not necessarily any riskier as business endeavors than, say, running a photocopy shop.

6. Howard Raiffa's book *Decision Analysis: Introductory Lectures on Choices Under Uncertainty* (New York: McGraw-Hill, 1997) does an extensive analysis of variations on the following basic problem.

   There are 1,000 urns. Eight hundred of them are of type U_1; each of these contain four red balls and six black balls. The remaining two hundred are of type U_2; each of these contain nine red balls and one black ball. One of these 1,000 urns is chosen at random and placed in front of you; you cannot identify its type or see the balls inside it. Which one of the following options maximizes your expected value, and what is that expected value?

   Option 1 Guess that the urn is of type U_1. If you are correct, you win \$40.00. Otherwise, you lose \$20.00.

   Option 2 Guess that the urn is of type U_2. If you are correct, you win \$100.00. Otherwise, you lose \$5.00.

   Option 3 Refuse to play the game.

   Your expected value from Option 1 is .8(40)+.2(-20) = 28. Your expected value from Option 2 is .8(-5)+.2(100) = 16. Your expected value from Option 3 is 0. So Option 1 maximizes your expected value.

7. *Challenge* (One variation) Prior to choosing one of the three options described above, you can conduct *at most one* of the following investigations.

(Note that you can also choose not to conduct any of these.) What strategy maximized your expected value, and what is that expected value?

Investigation 1 For a payment of \$8.00, you can draw a single ball at random from the urn.

Investigation 2 For a payment of \$12.00, you can draw two balls from the urn.

Investigation 3 For a payment of \$9.00, you can draw a single ball from the urn, and then (after looking at it) decide whether or not you want to pay \$4.50 to draw another ball. (Whether or not you want to replace the first ball before drawing the second is up to you.)

This is a difficult problem. For more information on it, read Raiffa's book or do some research on Bayes's Rule.

8. You're a bidder in an auction for an antique vase. If you lose the auction, you get nothing. If you win the auction, assume that your gain is the difference between the maximum amount you'd be willing to pay for the vase (say, \$100) and the actual amount that you end up paying. (So if you pay \$80, your gain is \$20.)

   (a) In a **first-price sealed bid auction**, you write down your bid b on a piece of paper and submit it to the auctioneer in a sealed envelope. After all the bids have been submitted, the auctioneer opens the envelopes and finds the highest bidder. That bidder gets the item, and pays a price equal to their bid. If the probability of winning with a bid of b is $\Pr(b)$, write down an expected value calculation for this auction.

   Your expected value from bidding b in either type of auction is

   $$\text{Prob}(b \text{ wins}) \cdot \text{Value}(b \text{ wins}) + \text{Prob}(b \text{ loses}) \cdot \text{Value}(b \text{ loses}).$$

   In a first-price auction, Value(b wins)$= 100 - b$ and Value(b loses)$= 0$; so your expected value is

   $$\text{Prob}(b \text{ wins}) \cdot (100 - b) + 0.$$

   (b) In a **second-price sealed bid auction**, everything's the same except that the winning bidder (the person with the highest bid) pays a price equal to the *second-highest* bid. Write down an expected value calculation for this auction if the probability of winning with a bid of b is $\Pr(b)$ and the highest bid *less* than b is c.

   In a second-price auction, Value(b wins)$= 100 - c$, where c is the highest bid less than b, and Value(b loses)$= 0$. So your expected value is

   $$\text{Prob}(b \text{ wins}) \cdot (100 - c) + 0.$$

(c) *Challenge* From your answers above, can you figure out what kind of strategy (i.e., what bid b) will maximize your expected value in the different auctions? In particular: should you bid your true value, $b = \$100$, or should you bid more or less than your true value? (We'll study auctions more in Chapter 10.)

Chapter 10 discusses this in more detail.

Chapter 3: Optimization over Time

1. Say you have \$100 in the bank today.

   (a) How much will be in the bank after 30 years if the interest rate is 5%? Call this amount y.

   Plug \$100, 5%, and 30 years into the future value of a lump sum formula to get $y \approx \$432.19$.

   (b) What is the present value of receiving y after 30 years? Call this amount z.

   Plug \$432.19, 5%, and 30 years into the present value of a lump sum formula to get $z \approx \$100$.

   (c) How does z compare with the \$100 you currently have in the bank? Can you explain why by looking at the relationship between the formulas for the present value and future value of lump sums?

   They are equal. The explanation here is that the formulas for present value and future value of lump sums are inverses of each other in that you can rearrange either equation to get the other:

$$\text{PV} = \frac{FV}{(1+r)^n} \Longleftrightarrow \text{FV} = (PV)(1+r)^n.$$

2. Intuitively, how much difference do you think there is between an annuity paying \$100 each year for 1 million years and a perpetuity paying \$100 each year forever? Can you mathematically confirm your intuition by relating the annuity formula to the perpetuity formula?

   Intuitively, there shouldn't be much difference. Mathematically, the annuity formula approaches the perpetuity formulas as n approaches infinity:

$$\lim_{n\to\infty} x \left[\frac{1 - \dfrac{1}{(1+r)^n}}{r} \right] = \frac{x}{r}.$$

3. Explain the perpetuity formula in terms of "living off the interest".

   The perpetuity formula says that the present value of a perpetuity paying x every year is $\frac{x}{r}$. This is like living off the interest because if you put

$\frac{x}{r}$ in the bank, every year you will get interest of $r \cdot \frac{x}{r} = x$, so with this principal you can finance annual consumption of x forever.

4. Consider a "\$20 million" lottery payoff paying \$1 million at the end of each year for 20 years.

   (a) Calculate the present value of this payoff if the interest rate is 5%.

   Plug \$1 million, 5%, and 20 years into the annuity formula to get about \$12.5 million as the present value of the annuity.

   (b) Calculate the present value of the related perpetuity paying \$1 million at the end of each year forever.

   Plug \$1 million and 5% into the perpetuity formula to get \$20 million as the present value of the perpetuity. Note that the extra payments you get—\$1 million annually beginning in year 21—are only worth about \$7.5 million in present value terms!

   (c) Assume that the lump sum payoff for your \$20 million lottery is \$10 million, i.e., you can opt to get \$10 million now instead of \$1 million at the end of each year for 20 years. Using trial and error, estimate the interest rate r that makes the lump-sum payoff for the lottery equal to the annuity payoff.

   Increasing r will make the lump sum payment more attractive, and decreasing r will make the annual payments more attractive. Trial and error yields $r \approx .075$ as the interest rate that makes the two payoffs equal in present value terms.

   (d) Calculate the present value of winning \$1 million at the *beginning* of each year for 20 years. Again, assume the interest rate is 5%. *Hint:* there are easy ways and hard ways to do this!

   The hard way to do this is to just calculate the present value of each payment and then add them all together. Easy way #1 is to realize that the difference between the end-of-year payments and the beginning-of-year payments is just an extra payment at the beginning of the first year and a lost payment at the end of the 20th year. The present value of \$1 million today is \$1 million, and the present value of \$1 million at the end of 20 years is \$380,000. Their difference is \$620,000, so adding this to the answer from (a) yields \$13.08 million. Easy way #2 is to see that the answer from (a) is the right answer from the perspective of one year ago, so using the future value of a lump sum formula to push this answer one year into the future gives us $\$12.46(1.05) = \$13.08$ million.

5. (The Rule of 72): A rule of thumb is that if you have money in the bank at r% (e.g., 10%), then your money will double in $\dfrac{72}{r}$ years, e.g., 7.2 years for a 10% interest rate.

(a) How many years will it actually take your money to double at 10%? (You can find the answer—plus or minus one year—through trial-and-error; if you know how to use logarithms, you can use them to get a more precise answer.) Compare the true answer with the Rule of 72 approximation.

Note: The elements of this problem featuring logarithms are not fair game for the exam. Having said that: Solving $2x = x(1.10)^t$ using logarithms yields $t = 7.27$, pretty close to the Rule of 72 estimate of 7.2.

(b) Do the same with a 5% interest rate and a 100% interest rate.

The Rule of 72 predicts 14.4 years at 5% and .72 years at 100%. The correct answer at 100% is 1 year (obviously, since the interest rate is 100%), so the Rule of 72 is not such a good approximation. The correct answer at 5% comes from solving $2x = x(1.05)^t$ using logarithms. We get $t \approx 14.2$, which is quite accurate.

(c) Do your results above suggest something about when the Rule of 72 is a good approximation?

They suggest that the Rule of 72 works well for small interest rates, but not for large ones.

6. Investment #1 pays you \$100 at the end of each year for the next 10 years. Investment #2 pays you nothing for the first four years, and then \$200 at the end of each year for the next six years.

(a) Calculate the present value of each investment if the interest rate is 5%. Which one has a higher present value?

For Investment #1, use the annuity formula to get a present value of about \$772 at a 5% interest rate. For Investment #2, the brute force way to calculate the present value is to calculate the present value of each of the 6 lump sum payments and then add them up to get about \$835. A more elegant way is to note that Investment #2 is equivalent to a ten-year annuity *minus* a four-year annuity. You can therefore use the annuity formula to calculate the present value of the ten-year annuity (\$1,544) and the four-year annuity (\$709). Subtracting one from the other gives a present value for Investment #2 of \$835.

Investment #2 is the better option at a 5% interest rate.

(b) Which investment has the greater present value at an interest rate of 15%?

Following the same process described above, Investments #1 and #2 have present values of \$502 and \$433, respectively. So Investment #1

(c) Do higher interest rates favor investment #1 or #2? Can you explain why using intuition and/or math?

Higher interest rates favor Investment #1, in which you essentially forfeit money in the distant future in order to get more money in the immediate future. Since higher interest rates make the future less important, they make Investment #1 more attractive.

(d) Can you think of any real-life decisions that have features like these?

There are many real-life decisions with similar features; for example, Investment #1 could be going to work right out of high school, and Investment #2 could be going to college for 4 years first to increase your earnings potential.

7. *Fun* Compound interest has been called the Eighth Wonder of the World. Here's why.

(a) Legend has it that the inventor of chess showed his game to the King of India (or maybe it was the Emperor of China), and the king was so impressed he offered the inventor a reward. The inventor said that all he wanted was one grain of rice for the first square of the board, two for the second, four for the third, and so on, doubling on each of the 64 squares of the chessboard. This seemed reasonable enough, so the king agreed. How many grains of rice was the king supposed to deliver for the 64th square?

The king was supposed to deliver $2^{63} \approx 1,000,000,000,000,000,000,000$ grains of rice for the last square.

(b) On Day 1, somebody dropped a lily pad in a mountain pond. The number of lily pads (and the percentage of pond they covered) doubled every day. On the 30th day, the pond was completely covered in lily pads. On which day was the pond half-covered?

The pond was half-covered on the 29th day.

(c) How tall do you think a piece of paper would be if you could fold it in half again and again and again, 40 times? Estimate the thickness of a piece of paper and then calculate this height.

The answer is the height of a single sheet of paper multiplied by $2^{39} \approx 1,000,000,000,000$. If 1,000 pages makes an inch, then this gives us 1,000,000,000 inches, or about 83 million feet, or about 16,000 miles.

Comment: These examples involve interest rates of 100% (i.e., doubling), but you will get similar results with much smaller interest rates as long as your time horizons are long enough. This is because all interest rate problems share a common feature: constant doubling time. Put \$100 in the bank at 100% interest and it will double every year: \$200, \$400, \$800,... At 1% interest it will take 70 years to double to \$200, and every 70 years thereafter it will double again (to \$400, \$800,...). So if we call 70 years a *lifetime*, we see that an *annual* interest rate of 1% is equivalent to a *lifetime* interest rate of 100%. So 1% growth and 100% growth are different in degree but not in spirit.

8. *Fun* The Intergovernmental Panel on Climate Change reports that human activity (especially the burning of fossil fuels such as coal, oil, and natural gas) is warming the earth. (Note: With the exception of this fact, all of the numbers &etc in this question are completely made up.)

   (a) Assume that global warming will raise sea levels and increase the frequency of hurricanes, leading to damages of $1 trillion ($= 10^{12} = 1,000,000,000,000$) at the end of each year for the next seven years. What is the present value of that damage if the interest rate is 4%? [Note: If all the zeroes confuse you or your calculator, use $1,000,000 or $1,000 instead.]

   Using the annuity formula we get a present value of about $6 trillion.

   (b) Next, assume that the full damages you've calculated above will only occur with probability 1/3. With probability 1/3 the damages will be only half as big, and with probability 1/3 the damages will be zero. What is the expected value of the damage caused by global warming? [Note: If you didn't answer part 8a above, just assume for this part that the total damage is $1,000,000.]

   The expected damages are $\frac{1}{3}(6) + \frac{1}{3}(3) + \frac{1}{3}(0) \approx \$3$ trillion.

   (c) Next, assume that the hurricanes &etc won't happen for 100 years. Using an interest rate of 4%, take the expected damages you calculated in part 8b and compute the present value of having that amount of damage occur 100 years in the future. [Note: If you didn't answer part 8b, just assume for this part that the total damage is $1,000,000.]

   Plug $3 trillion into the present value of a lump sum formula to get a present value of $59 billion.

   (d) What would be the present value of those damages if they won't occur for 500 years?

   Using the present value of a lump sum formula, we get $9,130.

9. The U.S. Federal Reserve Board (a.k.a. the Fed) recently took steps to lower interest rates in the U.S.: in January 2001 the Federal Funds rate was 6.0%, and by June it was 4.0%. You'll learn a whole bunch more about the Fed in Econ 201 (and can get details about changes in the Federal Funds rate at http://www.federalreserve.gov/fomc/fundsrate.htm), but we can already understand something about Alan Greenspan and his mysterious ways.

   (a) Pretend (you might not have to) that you're a consumer. You're thinking about borrowing some money to buy a new TV set (i.e., you're thinking about buying on credit). Since you can now get a loan at a lower interest rate, does the Fed's action make it more likely or less likely that you'll decide to buy that TV set?

   You're more likely to buy the TV set.

(b) There are a lot of consumers out there, all thinking rationally, just like you. Is the Fed's action likely to increase or decrease the total amount of stuff that we buy?
Increase.

(c) Pretend that you're running a business. You have made a tidy profit of $100, and you're trying to decide whether to put that $100 in the bank or spend it on expanding your business. Does the Fed's action make spending that money on expanding your business more attractive or less attractive?
More attractive.

(d) There are a lot of CEOs out there, all thinking rationally, just like you. So will the Fed's action tend to increase or decrease business expansions and economic activity?
Increase.

(e) Pretend that you're an investment guru. Somebody's unsuspecting grandmother has given you $100 to invest on her behalf, and you're trying to decide whether to invest that money in the bank or buy stock. Does the Fed's action make investing in the stock market more attractive or less attractive?
More attractive.

(f) There are a lot of investment gurus out there, all thinking rationally, just like you. So is the Fed's action likely to make the stock market go up or down?
Up.

10. *Fun* Here is some information from the National Archives:

> In 1803 the United States paid France $15 million ($15,000,000) for the Louisiana Territory—828,000 square miles of land west of the Mississippi River. The lands acquired stretched from the Mississippi River to the Rocky Mountains and from the Gulf of Mexico to the Canadian border. Thirteen states were carved from the Louisiana Territory. The Louisiana Purchase nearly doubled the size of the United States, making it one of the largest nations in the world.

At first glance, paying $15 million for half of the United States seems like quite a bargain! But recall that the Louisiana Purchase was almost 200 years ago, and $15 million then is not the same as $15 million now. Before we can agree with General Horatio Grant, who told President Jefferson at the time, "Let the Land rejoice, for you have bought Louisiana for a song," we should calculate the present value of that purchase. So: it has been about 200 years since the Louisiana Purchase. If President Jefferson had not completed that purchase and had instead put the $15 million in a bank account, how much would there be after 200 years at an interest rate of:

(a) 2%?

Plug \$15 million, 200 years, and 2% into the future value of a lump sum formula to get a current bank account balance of \$787 million.

(b) 8%? (See problem 5 on page 15 for more about the Louisiana Purchase.)

Plug \$15 million, 200 years, and 8% into the future value of a lump sum formula to get a current bank account balance of \$72.6 trillion

11. It is sometimes useful to change interest rate time periods, i.e., to convert a monthly interest rate into a yearly interest rate, or vice versa. As with all present value concepts, this is done by considering money in the bank at various points of time.

(a) To find an annual interest rate that is approximately equal to a monthly interest rate, multiply the monthly interest rate by 12. Use this approximation to estimate an annual interest rate that is equivalent to a monthly rate of 0.5%.

The approximate annual interest rate is 6%.

(b) Assume you have \$100 in the bank at a monthly interest rate of 0.5%. How much money will actually be in the bank at the end of one year (12 months)? Use your answer to determine the actual annual interest rate that is equivalent to a monthly rate of 0.5%.

Use the future value of a lump sum formula to calculate how much money we'll have at the end of 12 months if we put \$100 in the bank at a monthly interest rate of 0.5%, i.e. $r = 0.005$: FV= $\$100(1.005)^{12} \approx$ 106.17. So a monthly interest rate of 0.5% actually corresponds to an annual interest rate of 6.17%.

(c) To find a monthly interest rate that is approximately equal to an annual interest rate, divide the annual interest rate by 12. Use this approximation to estimate a monthly interest rate that is equivalent to an annual rate of 6%.

The approximate monthly interest rate is 0.5%.

(d) *Challenge* Use logarithms to determine the actual monthly interest rate that is equivalent to an annual rate of 0.5%.

Use the future value of a lump sum formula to set $x(1+r)^{12} = x(1.06)$ and solve for r using logarithms. This yields $r \approx .00487$, i.e., an interest rate of about 4.87%.

12. *Fun* (Thanks to Kea Asato.) The 2001 Publishers Clearing House "\$10 million sweepstakes" came with three payment options:

Yearly Receive \$500,000 immediately, \$250,000 at the end of Year 1 and every year thereafter through Year 28, and \$2.5 million at the end of Year 29 (for a total of \$10 million).

Weekly Receive \$5,000 at the end of each week for 2,000 weeks (for a total of \$10 million).

Lump sum Receive \$3.5 million immediately.

Calculate the present value of these payment options if the interest rate is 6% per year. What is the best option in this case? (See problem 6 on page 15 for more about this problem.)

To calculate the present value of the yearly option, we need to split the payment into three parts: the \$500,000 you get immediately (which has a present value of \$500,000), the 28-year annuity (which, using the annuity formula, has a present value of about \$3.35 million), and the payment in year 29 (which, using the present value of a lump sum formula, has a present value of about \$460,400). We add these up to get a present value of about \$4.31 million.

To calculate the present value of the weekly option, we first need to calculate a weekly interest rate that is in accordance with a yearly interest rate of 6%. A decent approximation is to divide the yearly interest rate by the number of weeks in a year (52), yielding a weekly interest rate of about 0.115%, i.e., $r \approx 0.00115$.[2] Using this value of r in the annuity formula along with \$5,000 and 2,000 weeks gives us a present value of about \$3.91 million for the weekly option.

Finally, the present value of receiving \$3.5 million immediately is \$3.5 million. So the best option is the yearly payments.

Chapter 4: More Optimization over Time

1. If a bank is paying 14.4% and inflation is 8%, calculate the real interest rate. Round to the nearest .1% (Hint: Think about purchasing power relative to a good whose price increases at the rate of inflation.) Use both the true formula and the approximation and compare them.

   The approximation is $14.4\% - 8\% = 6.4\%$. The actual answer comes from the formula, and gives a result of 5.9%.

2. Explain (as if to a non-economist) why the formula relating real and nominal interest rates makes sense. (Hint: Recall that the real interest rate measures increases in purchasing power, and think about how much more of some good you'll be able to purchase in one year if your bank account pays the nominal interest rate and the good's prices increases with inflation.)

   This is explained in the text.

[2]If you don't like approximations, here's how to use logarithms to get the true value. If you put x in the bank at a weekly interest rate of $100 \cdot r\%$, after one year (52 weeks), you'll have $x(1+r)^{52}$ in the bank. We want this to equal $x(1.06)$, since this follows from a yearly interest rate of 6%. Setting $x(1+r)^{52} = x(1.06)$ and solving for r using logarithms gives us $r \approx .00112$, i.e., an interest rate of about 0.112%.

3. Assume that the nominal interest rate is 10% per year and that the rate of inflation is 5% per year. Round all your answers as appropriate.

   (a) You put \$100 in the bank today. How much will be in your account after 10 years?

   Use the nominal interest rate and the future value formula to get a bank account balance of about \$259.37.

   (b) You can buy an apple fritter (a type of donut) for \$1 today. The price of donuts goes up at the rate of inflation. How much will an apple fritter cost after 10 years?

   Use the inflation rate and the future value formula to get an apple fritter price of about \$1.63.

   (c) Calculate x, the number of apple fritters you could buy for \$100 today. Then calculate y, the number of apple fritters you could buy after ten years if you put that \$100 in the bank. Finally, calculate $z = 100 \cdot \frac{y - x}{x}$. (The deal with z is that you can say, "If I put my money in the bank, then after ten years I will be able to buy z% more apple fritters.")

   Today you have \$100 and fritters cost \$1, so you can buy $x = 100$ of them. In ten years you'll have \$259.37 and fritters will cost \$1.63, so you'll be able to buy about $y = 159$ of them. So we can calculate $z \approx 59$.

   (d) Given the nominal interest rate and inflation rate above, calculate the real interest rate to two significant digits (e.g., "3.81%"). Check your answer with the "rule of thumb" approximation.

   The rule of thumb approximation says that the real interest rate should be about $10\% - 5\% = 5\%$. The actual value is $\frac{1+.1}{1+.05} - 1 \approx .048$, i.e., 4.8%.

   (e) Calculate how much money you'd have after 10 years if you put \$100 in the bank today *at the real interest rate you calculated in the previous question* (3d). Compare your answer here with the result from question 3c.

   If you put \$100 in the bank at this interest rate, after 10 years you'd have about \$159. So you get $z = 59$ as your gain in purchasing power.

4. Here are a couple of rules of thumb concerning the use of real (rather than nominal) interest rates in present value calculations.

   Use real when your payments are inflation-adjusted. Somebody offers to sell you a lemon tree that will bear 100 lemons at the end of each year. The price of lemons is \$1.00/lemon right now, and will rise at the rate of inflation, which is 4% per year; the nominal interest rate is 6%.

(a) What is the present value of the lemon tree if it will bear fruit for 5 years and then die?
Use the annuity formula and the real interest rate (about $6-4 = 2\%$) to get a present value of about \$470.

(b) What if it will bear fruit forever?
Use the perpetuity formula and the real interest rate (about $6 - 4 = 2\%$) to get a present value of about \$5,000.

Use real to calculate future purchasing power. You and a buddy win the \$20 million grand prize in a lottery, and you choose to accept a lump sum payment of \$10 million, which you divide equally. Your buddy immediately quits school and moves to Trinidad and Tobago. Being more cautious, you put your \$5 million in a 40-year CD paying 6%, figuring that after 40 years your wealth will have increased 10-fold and so you'll be able to buy 10 times more stuff. Are you figuring correctly if the inflation rate is 4%? You're not figuring correctly because you're forgetting that prices are going to rise. Yes, you'll have 10 times more money, but you won't be able to buy 10 times more stuff. Using the real interest rate and the future value formula, we get a future value of \$11 million, or about 2.2 times more purchasing power.

5. *Fun* Recall the question from Chapter 3 concerning the Louisiana Purchase. That problem (#10 on page 11) asked you to calculate the present value of the \$15 million President Jefferson spent in 1803 to buy the Louisiana Territory from France, using interest rates of 2% and 8%. Assume now that 2% was the real interest rate over that time period and that 8% was the nominal interest rate over that time period. Which is the correct interest rate to use?

   Since we're trying to figure out what the current bank account balance is and the bank pays the nominal interest rate, we should use the nominal interest rate to determine if the Louisiana Purchase was really a great deal.

   Note that an estimate of 2% for the real interest rate actually does make sense; in fact, it's called the Fischer Hypothesis, which you might have (or may yet) come across in macroeconomics. The 8% figure for the nominal interest rate, in contrast, is entirely fictitious; you'll have to study some economic history if you want a real approximation for the nominal interest rate over the last 200 years.

6. *Fun* Recall the question from Chapter 3 concerning the Publishers Clearing House sweepstakes. That problem (#12 on page 12) asked you to calculate the present value of different payment options. In calculating those present values, should you use the real interest rate or the nominal interest rate?

We're trying to figure out how much money we need to put in the bank today in order to finance cash payments in the future. Since the bank pays the nominal interest rate, that's the rate we should use.

Chapter 5: Transition: Arbitrage

1. Explain (as if to a non-economist) why comparable investments should have comparable expected rates of return.

   This is answered (to the best of my abilities) in the text.

2. Consider a company that has a 10% chance of going bankrupt in the next year. To raise money, the company issues junk bonds paying 20%: if you lend the company \$100 and they *don't* go bankrupt, in one year you'll get back \$120. Of course, if the company *does* go bankrupt, you get nothing. What is the *expected* rate of return for this investment? If government bonds have a 3% expected rate of return, what is the **risk premium** associated with this investment?

   If you lend the company \$100, the expected value of your investment is $.90(120) + .10(0) = \$108$, meaning that your expected rate of return is 8%. If government bonds have a 3% expected rate of return, the risk premium associated with the junk bonds is $8\% - 3\% = 5\%$.

3. Economic reasoning indicates that comparable investments should have comparable expected rates of return. Do the following examples contradict this theory? Why or why not?

   (a) Microsoft stock has had a much higher rate of return over the last twenty years than United Airlines stock.

   The *actual* rates of return turned out to be different, but it could still be true that at any point in time over the last twenty years the *expected* rates of return were the same. Economic reasoning suggests that they should have been; otherwise investors would not have been acting optimally. In terms of its effect on this logic, the fact that the actual rates of return turned out to be different is no more cause for concern than the fact that some lottery tickets turn out to be winners and some turn out to be losers.

   (b) Oil prices are not going up at the rate of interest.

   Again, actual rates of return can be different even though expected rates of return are the same at any moment in time. For example, there may be unexpected advances (or setbacks) in the development of electric cars or other alternatives to gasoline-powered cars.

   An alternative explanation is that the cost of extracting oil has gone down over time; as a result, the profits from each barrel of oil may be going up at the rate of interest even though the price of oil may not steady or falling.

(c) Junk bonds pay 20% interest while U.S. Treasury bonds pay only 4% interest.

Companies that issue junk bonds are financially troubled; if they go bankrupt, you lose both the interest and the principal. So even though junk bonds may offer 20% interest, their expected rate of return is much less than 20%, and therefore much closer to the expected rate of return of U.S. Treasury bonds. Also, the U.S. Treasury isn't likely to go bankrupt, meaning that two assets don't have comparable risk. So it is likely that there is a risk premium associated with the junk bonds.

(d) Banks in Turkey (even banks that have no risk of going bankrupt) pay 59% interest while banks in the U.S. only pay 4% interest.

The answer here is that the rate of inflation is much higher in Turkey than in the U.S. So even though the two banks pay different nominal interest rates, their expected real interest rates may be equal.

One way to see the effect of inflation is to imagine that you'd invested 100 U.S. dollars in a Turkish bank in June 2000. The exchange rate then was about 610,000 Turkish lira to $1, so your $100 would have gotten you 61 million Turkish lira. After one year at a 59% interest rate, you would have 97 million lira. But when you try to change that back into U.S. dollars, you find that the exchange rate has changed: in June 2001 it's 1,240,000 lira to $1, so your 97 million lira buys only $78.23. You actually would have lost money on this investment!

Answer Key for Part II: One v. One, One v. Many

Chapter 6: Cake-Cutting

1. "Differences in opinion make fair division harder." Do you agree or disagree? Explain why.

   Arguably, differences in opinion make fair division easier, not harder. For example, if one child only likes vanilla and the other child only like chocolate, then cake division is, well, a piece of you-know-what.

2. *Challenge* Can you convince yourself—or someone else—that each child can get at least $\frac{1}{n}$th of the cake (in his or her estimation) with the divide-and-choose algorithm? How about with the moving-knife algorithm?

   This question is not fair game for exam, but is solvable via induction.

3. Explain (as if to a non-economist) the Coase Theorem and its implications for the cake-cutting problem. (In other words, explain why the economist's solution to the cake-cutting problem hinges on allowing the children to trade after the initial allocation has been made.)

   The Coase Theorem says that people who are free to trade have a strong incentive to trade until they exhaust all possible gains from trade, i.e., until they complete all possible Pareto improvements and therefore reach a Pareto efficient allocation of resources. The implication for the cake-cutting problem is that a "mom" whose sole concern is efficiency can divide the cake up however she wants—as long as the children can trade, they should be able to reach a Pareto efficient allocation regardless of the starting point. For example, if you give the chocolate piece to the kid who loves vanilla and the vanilla piece to the kid who loves chocolate, they can just trade pieces and will end up at a Pareto efficient allocation.

4. *Challenge* Show that an envy-free division is also proportional.

   To show that envy-free implies proportional, we will show that not proportional implies not envy-free. If a cake division is not proportional, one child gets less than $\frac{1}{n}$th of the cake (in her estimation). This means that (according to her estimation) the other $(n-1)$ children get more than $\frac{n-1}{n}$th of the cake. Therefore at least one of the other children must have a piece bigger than $\frac{1}{n}$th, meaning that the cake division is not envy-free.

5. (Specialization and Gains from Trade) In this chapter we're examining the mechanisms of trade and the benefits of allowing people to trade. Here is one (long, but not difficult) numerical example about trade, based on what is sometimes called the **Robinson Crusoe model** of an economy.

   Imagine that Alice and Bob are stranded on a desert island. For food, they must either hunt fish or gather wild vegetables. Assume that they each have 6 hours total to devote to finding food each day, and assume that they really like a balanced diet: at the end of the day, they each want to have equal amounts of fish and vegetables to eat. We are going to examine the circumstances under which they can gain from trade.

   Story #1: Imagine that Alice is better than Bob at fishing (she can catch 2 fish per hour, and he can only catch 1 per hour) and that Bob is better than Alice at gathering wild vegetables (he can gather 2 per hour, and she can only gather 1). Economists would say that Alice has an **absolute advantage** over Bob in fishing and that Bob has an absolute advantage over Alice in gathering vegetables. Intuitively, do you think they can gain from trade? (Just guess!) Now, let's find out for sure:

   (a) If Alice and Bob could not trade (e.g., because they were on different islands), how many hours would Alice spend on each activity, and how much of each type of food would she end up with? How many hours would Bob spend on each activity, and how much of each type of food would he end up with? (Hint: Just play with the numbers, remembering that they each have six hours and want to get equal amounts of fish and vegetables.)

   Alice would spend 4 hours gathering veggies and 2 hours fishing, providing her with 4 veggies and 4 fish. Bob would do exactly the opposite (4 hours fishing, 2 hours gathering veggies) and would also end up with 4 of each.

   (b) Now, imagine that Alice and Bob can trade with each other. Consider the following proposal: Alice will specialize in fishing, and Bob will specialize in gathering vegetables. After they each devote six hours to their respective specialties, they trade with each other as follows: Alice gives half her fish to Bob, and Bob gives half his vegetables to Alice. How many fish and how many vegetables will they each end up with in this case?

   If they specialize, Alice spends 6 hours fishing, so she gets 12 fish; Bob spends 6 hours hunting, so he gets 12 veggies. Then they split the results, so each gets 6 fish and 6 veggies, a clear Pareto improvement over the no-trade situation.

   (c) Is this a Pareto improvement over the no-trade result in question 5a?

Yes.

Story #2: Now, imagine that Alice is better than Bob at fishing (she can catch 6 fish per hour, and he can only catch 1 per hour) and that Alice is also better than Bob at gathering wild vegetables (she can gather 3 per hour, and he can only gather 2). Economists would say that Alice has an absolute advantage over Bob in both fishing and gathering vegetables. Intuitively, do you think they can gain from trade? (Just guess!) Now, let's find out for sure:

(d) If Alice and Bob could not trade (e.g., because they were on different islands), how many hours would Alice spend on each activity, and how much of each type of food would she end up with? How many hours would Bob spend on each activity, and how much of each type of food would he end up with?

They would allocate their time as before, but now Alice would get 12 fish and 12 veggies and Bob would get 4 fish and 4 veggies.7

(e) Now, imagine that Alice and Bob can trade with each other. Consider the following proposal: Alice will specialize in fishing, increasing the amount of time that she spends fishing to 3 hours (leaving her with 3 hours to gather vegetables); and Bob will specialize in gathering vegetables, increasing the amount of time that he spends gathering vegetables to 5 hours (leaving him 1 hour to fish). After they each devote six hours as described above, they will trade with each other as follows: Alice gives 5 fish to Bob, and Bob gives 4 vegetables to Alice. How many fish and how many vegetables will they each end up with in this case?

If they specialize as described in the problem, Alice ends up with 18 fish and 9 veggies, and Bob ends up with 1 fish and 10 veggies. After they trade, Alice ends up with 13 fish and 13 veggies, and Bob ends up with 6 fish and 6 veggies, another Pareto improvement!

(f) Is this a Pareto improvement over the no-trade result in question 5d?

Yes.

Now, forget about possible trades and think back to Alice and Bob's productive abilities.

(g) What is Alice's cost of vegetables in terms of fish? (In other words, how many fish must she give up in order to gain an additional vegetable? To figure this out, calculate how many minutes it takes Alice to get one vegetable, and how many fish she could get in that time. Fraction are okay.) What is Alice's cost of fishing in terms of vegetables?

Alice must give up 2 fish to get one vegetable, and must give up 0.5 veggies to get one fish.

(h) What is Bob's cost of fishing in terms of vegetables? What is Bob's cost of vegetables in terms of fish?

Bob must give up 0.5 fish to get one vegetable, and 2 veggies to get one fish.

(i) In terms of vegetables, who is the least-cost producer of fish?

Alice.

(j) In terms of fish, who is the least-cost producer of vegetables?

Bob.

When they concentrate on the items for which they are the least-cost producer, they can both benefit from trade even though Alice has an absolute advantage over Bob in both fishing and gathering veggies. This is the concept of comparative advantage.

The punch line: Having each party devote more time to their least-cost product is the concept of **comparative advantage**.

Chapter 7: Economics and Social Welfare

1. Explain (as if to a non-economist) the following concepts, and use each in a sentence.

(a) Pareto inefficient

Pareto inefficient means that it's possible to make someone better off without making anyone else worse off; in other words, there's a "free lunch". Example: It is Pareto inefficient to give Tom all the chicken and Mary all the veggies because Tom's a vegetarian and Mary loves chicken.

(b) Pareto improvement

A Pareto improvement is a reallocation of resources that makes one person better off without making anyone else worse off. Example: giving Tom the veggies and Mary the chicken is a Pareto improvement over giving Tom the chicken and Mary the veggies.

(c) Pareto efficient

Pareto efficient means that there is no "free lunch", i.e., it's not possible to make someone better off without making anyone else worse off. Example: Giving Tom the veggies and Mary the chicken is a Pareto efficient allocation of resources.

2. "A Pareto efficient outcome may not be good, but a Pareto inefficient outcome is in some meaningful sense bad."

(a) Give an example or otherwise explain, as if to a non-economist, the first part of this sentence, "A Pareto efficient outcome may not be good."

A Pareto efficient allocation of resources may not be good because of equity concerns or other considerations. For example, it would be Pareto efficient for Bill Gates to own everything (or for one kid to get the whole cake), but we might not find these to be very appealing resource allocations.

(b) Give an example or otherwise explain, as if to a non-economist, the second part of this sentence, "A Pareto inefficient outcome is in some meaningful sense bad."

A Pareto inefficient allocation is in some meaningful sense bad because it's possible to make someone better off without making anybody else worse off, so why not do it?

3. "Any Pareto efficient allocation is better than any Pareto inefficient allocation."

(a) How would an economist define "better"? Can you add anything else to that definition?

Economists would use the concept of Pareto improvement to define "better". (Note that this does not include equity considerations, e.g., the idea that cutting the cake 50-50 might be "better" than giving one child the whole cake.)

(b) Assume that we're using the economist's definition of "better". Do you agree with this claim? If so, explain. If not, provide a counterexample or otherwise explain.

The claim that any Pareto efficient allocation is a Pareto improvement over any Pareto inefficient allocation is not true. For example, giving one child the whole cake is a Pareto efficient allocation, and giving each child one-third of the cake and throwing the remaining third away is Pareto inefficient, but the former is not a Pareto improvement over the latter.

4. Many magazines and newspapers have on-line archives containing articles from past issues. As with many information commodities, is costs essentially nothing for the publisher to provide readers with archived material. But in many cases they charge a fee (usually about $1) to download an article from the archive.

(a) Sometimes the maximum I'm willing to pay is $.25, so instead of downloading it I just do without. Is this outcome efficient? If not, identify a Pareto improvement.)

No. One Pareto improvement would be for the publisher to provide me with the article for free: I'm better off, and they're not any worse off. Another Pareto improvement is for me to pay $.25 for the article: I'm not any worse off, and the publisher is better off by $.25.

(b) Imagine that the publisher could engage in perfect price discrimination, i.e., could figure out the maximum amount that each reader is willing to pay for each article, and then charge that amount. Would the result be efficient?

Yes. Perfect price discrimination by a monopolist is Pareto efficient because it's not possible to make any of the customers better off without making the monopolist worse off.

(c) Explain briefly why the outcome described in question 4b is unlikely to happen.

The monopolist would need detailed information about all its different customers, e.g., the maximum amount each customer is willing to pay for each different article; such information is not readily available. There might also be a problem with resale, i.e., with some individuals paying others to purchase articles for them.

Chapter 8: Sequential Move Games

1. *Challenge* Explain (as if to a non-economist) why backward induction makes sense.

   This is explained (to the best of my abilities) in the text. The basic idea is that you need to anticipate your rival's response.

2. Consider tipping, a social phenomenon observed in some (but not all!) countries in which restaurant patrons leave some extra money behind for their waiter or waitress. Would tipping provide much of an incentive for good service if the tips were handed over at the beginning of the meal rather than at the end? Are there any difficulties in the incentive structure when tips are left at the end of meals? Write down game trees to support your arguments.

   Tipping at the beginning of the meal is problematic because then the waitress has no incentive to provide good service. (The tip is already sunk.) Tipping at the end of the meal is problematic because then the customer has no incentive to provide the tip. (The service is already sunk.)

3. (Overinvestment as a barrier to entry) Consider the following sequential move games of complete information. The games are between an incumbent monopolist (M) and a potential entrant (PE). You can answer these questions without looking at the stories, but the stories do provide some context and motivation.

   Story #1 (See figure A.1): Firm M is an incumbent monopolist. Firm PE is considering spending $30 to build a factory and enter the market. If

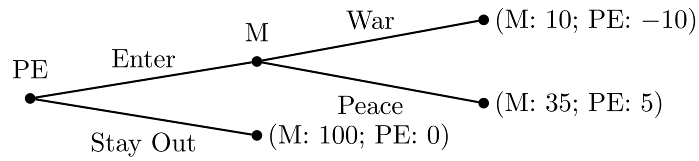


Figure A.1: Story #1

firm PE stays out, firm M gets the whole market. If firm PE enters the market, firm M can either build another factory and engage in a price war or peacefully share the market with firm PE.

(a) Identify (e.g., by circling) the likely outcome of this game.
Backward induction predicts an outcome of (M: 35, PE: 5).

(b) Is this outcome Pareto efficient? Yes No (Circle one. If it is not Pareto efficient, identify, e.g., with a star, a Pareto improvement.)
Yes.

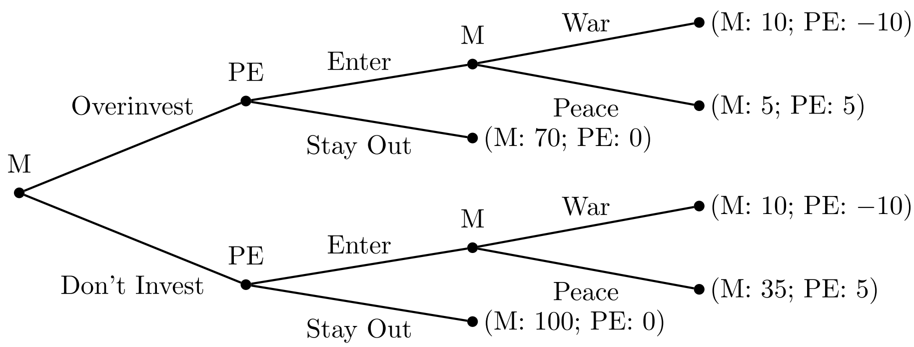


Figure A.2: Story #2

Story #2 (See figure A.2): The monopolist (firm M) chooses whether or not to overinvest by building a second factory for \$30 even though one factory is more than enough. Firm PE (the potential entrant) sees what firm M has done and decides whether to enter or stay out, and if PE enters then M decides whether or not to engage in a price war.

(a) Identify (e.g., by circling) the likely outcome of this game.

Backward induction predicts an outcome of (M: 70, PE: 0).

(b) Is this outcome Pareto efficient? Yes No (Circle one. If it is not Pareto efficient, identify, e.g., with a star, a Pareto improvement.)

No; a Pareto improvement is (M: 100, PE: 0).

4. (The Sticks Game) The sticks game works as follows: We put n sticks on the table. Beginning with Player 1, the two players take turns removing either one or two sticks. The player who removes the last stick must pay the other player \$1.

(a) If there are 10 sticks on the table, which player would you rather be, and what strategy will you employ? [Hint: Use backwards induction!]

If there are 10 sticks on the table, you should be player 2. Whenever your opponent takes 1 stick, you take 2; when he takes 2 sticks, you take 1. So you can force your opponent to move with 7 sticks, then 4 sticks, then 1 stick—so you win!

(b) *Challenge* If there are n sticks on the table, which player would you rather be? Can you describe a general strategy?

Hint: The above answer suggests a general strategy to follow.

(c) *Super Challenge* (Note: I haven't thought much about the second question here, and in any case haven't solved it.) Analyze this problem when there are m piles of sticks (each turn one of the players picks one of the piles and removes one or two sticks from it) or when there are more than two players (the losing player must pay \$1 to each of the others).

I only have a partial answer, so let me know if you solve all or part of this!

5. (The Ice Cream Pie Game, from Dixit and Nalebuff) Two players take turns making take-it-or-leave-it offers about the division of an ice cream pie. In the first round, the whole pie is available; if Player 2 accepts Player 1's proposal then the two players share the entire pie; if Player 2 rejects Player 1's proposal, half of the pie melts away, and we go to round two (in which Player 2 makes a take-it-or-leave-it offer about the division of the remaining pie). The game ends when an offer is accepted, or after the end of the nth period (at which point Mom eats the remaining pie, meaning that the players get nothing).

   (a) Predict the outcome of the game when there are 1, 2, and 3 periods.

   With one period, Player 1 offers Player 2 a sliver, and Player 2 accepts. With two periods, Player 1 offers Player 2 half the cake, and Player 2 accepts. (Both know that if Player 2 refuses, half the cake melts, Player 2 will offer Player 1 a sliver of the remaining half, and Player 1 will accept.) With three periods, Player 1 offers Player 2 one-quarter of the cake, and Player 2 accepts. (Both know that if Player 2 refuses, she'll have to offer Player 1 at least half of the remaining half, meaning that she'll get at most one-quarter.)

   (b) Now assume that 1/3rd of the pie (rather than 1/2) melts away each period. Predict the outcome when there are 1, 2, and 3 periods.

   With one period, Player 1 offers Player 2 a sliver, and Player 2 accepts. With two periods, Player 1 offers Player 2 two-thirds of the cake, and Player 2 accepts. (Both know that if Player 2 refuses, one-third of the cake melts, Player 2 will offer Player 1 a sliver of the remaining two-thirds, and Player 1 will accept.) With three periods, Player 1 offers Player 2 two-ninths of the cake, and Player 2 accepts. (Both know that if Player 2 refuses, she'll have to offer Player 1 at least two-thirds of the remaining two-thirds, meaning that she'll get at most two-ninths.)

   (c) Hopefully your prediction is that the first offer made is always accepted. Try to understand and explain (as if to a non-economist) why this happens.

This is the magic of the Coase Theorem. It is in neither player's interest to let the cake melt away, so they have a strong incentive to figure things out at the beginning and bring about a Pareto efficient outcome. You can see the same phenomenon at work in labor disputes and lawsuits, many of which get settled before the parties really begin to hurt each other.

6. Make up some game trees (players, options, payoffs, etc.) and solve them using backward induction. It may help to exchange games and solutions with a neighbor.

   I'd be happy to look over your work if you do this.

7. (The Draft Game, from Brams and Taylor) Three football teams (X, Y, Z) are involved in a draft for new players. There are six players to choose from (Center, Guard, Tailback, Quarterback, Halfback, Fullback), and the draft works as follows: First X chooses a player, then Y chooses one of the remaining five players, then Z chooses one of the remaining four players (this constitutes the first round of the draft); the same procedure is repeated in the second round, at the end of which all six players are taken.

   The teams' preferences are as follows:

	Top choice	Second	Third	Fourth	Fifth	Sixth
X	C	G	T	Q	H	F
Y	H	F	G	C	Q	T
Z	T	F	H	Q	C	G

   Assume that the teams all know each others' preferences. Then we can model the draft as a game tree, with team X choosing first &etc. The complete game tree for this draft is quite involved, but *trust me, it all boils down to the game tree shown in Figure A.3*.

   The payoffs for this game are the players each team gets. For example, (CG, HQ, TF) indicates that team X gets the Center and the Guard (its #1 and #2 choices), team Y gets the Halfback and the Quarterback (#1 and #2), and team Z gets the Tailback and the Fullback (#1 and #4). Clearly each team would prefer to get the players it likes the most, e.g., team X prefers CT (or TC) to CQ or GQ.

   (a) The "naive" strategy is for each team to choose its top choice among the available players every time it gets to pick. What is the outcome of this strategy?

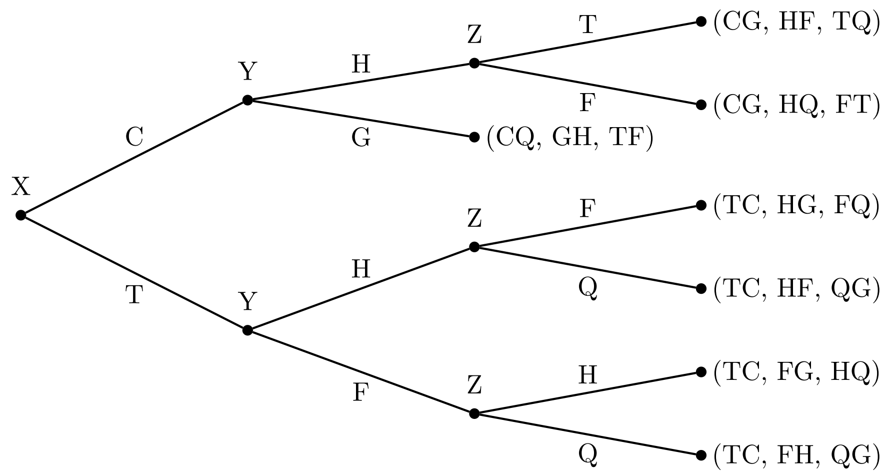


Figure A.3: The draft game

The naive outcome is for X to choose C, Y to choose H, and Z to choose T, producing the "naive outcome" at the top of the game tree.

(b) If teams X and Y pursue this naive strategy by picking C and H in the first round, should team Z also pursue this strategy, i.e., pick T? Briefly explain why or why not.

No. If X and Y choose C and H, Z will choose F because this produces a better outcome for Z: FT is better than TQ! (But now backward induction kicks in: Y anticipates this, and so Y will choose G instead of H—GH is better than HQ. But X anticipates this, and so knows that a choice of C will result in CQ. X then uses backward induction to solve the bottom half of the tree—Z will choose F in the top part and H in the lower part, so Y will choose H because HG is better than FG—and determine that a choice of T will result in TC. Since X prefers TC to CQ, X chooses T in the first round, leading Y to choose H and Z to choose F.

(c) What outcome do you expect from this game using backward induction?

Backward induction leads to a result of (TC, HG, FQ).

(d) Is the expected outcome you identified Pareto efficient? If so, explain. If not, identify a Pareto improvement.

This is not Pareto efficient: the "naive" strategies produce better outcomes for all three teams!

(e) Statement 1: "In the first round, the optimal move for each team is to pick the best available player." Statement 2: "In the second round, the optimal move for each team is to pick the best available player." Explain why Statement 1 is false but Statement 2 is true.

Statement #1 is false because each team's choice in the first round will have strategic implications for its options in the second round. Statement #2 is true because each team's choice in the second round has no further ramifications; since there are no more rounds, in the second round each team faces a simple decision tree.

(f) *Challenge* Prove that the game tree really does boil down to what's shown on the previous page.

This is a time-consuming problem. Thanks to K.B. for finding two strategies that yield this same outcome!

8. *Fun* (The Hold-Up Problem) In the movie *Butch Cassidy and the Sundance Kid* (1969), Paul Newman and Robert Redford play Wild West bank robbers who are particularly fond of robbing the Union Pacific Railroad. The CEO of the railroad, Mr. E. H. Harriman, hires a "superposse" of gunslingers to bring the two robbers to justice, dead or alive. After a long (and rather boring) chase scene, Butch and Sundance manage to escape. Afterwards, Butch reads about the superposse in the newspaper and has this to say:

> A set-up like that costs more than we ever took...That crazy Harriman. That's bad business. How long do you think I'd stay in operation if every time I pulled a job, it cost me money? If he'd just pay me what he's spending to make me stop robbin' him, I'd stop robbin' him. [Screaming out the door at E. H. Harriman:] Probably inherited every penny you got! Those inherited guys—what the hell do they know?

(a) Is what Harriman is doing bad business? Explain why or why not. You answer may depend on the assumptions you make, so explicitly state any and all assumptions. You might also want to draw a game tree, make up appropriate payoffs, and solve the game using backwards induction.

The answer here depends on your assumptions. See below for my take on it. . .

(b) "If he'd just pay me what he's spending to make me stop robbin' him, I'd stop robbin' him." Assume this statement is true. What does it say about the efficiency or inefficiency of the situation?

The situation is Pareto inefficient.

(c) What do you think about the argument contained in the previous quote? Can you see why this is called the "hold-up problem"?

The key issue here is that Butch Cassidy is a bank robber, and hence cannot be bound to contracts or other agreements. Sure, Harriman could pay him the money, but what guarantee does he have that this will make Butch stop robbing his train? A more likely outcome is that Butch will take the money and continue to rob the train, and then Harriman will be out even more money. So Harriman hires the superposse instead, even though both he and Butch would be better off with an alternative outcome.

(d) The hold-up problem also applies to students working jointly on projects and to firms engaged in joint ventures: after one member makes an irreversible investment, the other member may try to renegotiate the terms of the deal. Explain how contracts might help in preventing this difficulty, and why contracts wouldn't work in the case of Butch Cassidy.

Contracts can help by forcing players to act in certain ways; then the Coase them allows them to negotiate an efficient outcome. The Coase Theorem doesn't work in the case of Butch Cassidy because he's an outlaw: there's no way to bind an outlaw to an enforceable contract.

9. *Fun* The IgoUgo travel guide Andiamo provides the following advice for crossing the street in Rome: "First, stand towards a coffee bar and watch a local or two. See how they boldly walk out into traffic? Now it's your turn! Choose your moment but don't hesitate for too long. Waiting for traffic to clear will not happen. When you appear to have the most lead-time, step boldly off the curb and walk swiftly and confidently for the opposite side of the street. Do not look at the traffic careening towards you—believe it or not, they will stop for you! But do not look at them—do not make eye contact—this is an invitation for sport. Just walk briskly with your head up and your eyes on the prize- the opposite sidewalk."

(a) "[D]o not make eye contact—this is an invitation for sport." Explain.

If you make eye contact with the driver, the driver will pretend that she's not going to stop, and then you'll get scared and won't go for it, so then the driver *won't* stop.

(b) Set up a game tree for this "pedestrian in Rome" game and solve it.

The game tree here has you choosing to look or not look. If you choose not to look, the driver chooses to stop or not, and the payoffs are obvious. If you choose to look, the driver chooses to stop or not, and in each of those situations you must choose whether or not to push the issue.

10. (The Centipede Game) There are 6 \$1 bills on a table. Players 1 and 2 take turns moving. Each turn the player moving takes either \$1 (in which case it becomes the other player's turn) or \$2 (in which case the game ends). Each player wants to get as much money as possible.

(a) Draw a game tree for this game.

The game tree is pictured and described in the text.

(b) Predict the outcome of this game.

As discussed in the text, backward induction predicts that Player 1 will immediately choose \$2 and end the game, yielding an outcome of $(2, 0)$.

(c) Is this outcome Pareto efficient? If so, explain briefly. If not, identify a Pareto improvement.

No. There are many Pareto improvements, e.g., $(2, 2)$.

(d) *Challenge* Can you generalize this result when there are n bills on the table? (Hint: Try induction.)

You can do this with induction; this exercise also suggests why backward induction has the name it does.

(e) *Super Challenge* Can you reconcile the previous answer with your intuition about how this game might actually get played in real life?

This is a truly difficult philosophical question. If you're interested, there's an interesting chapter (and a great bibliography) on this topic, in the guise of "the unexpected hanging", in Martin Gardner's 1991 book, *The Unexpected Hanging, and Other Mathematical Diversions*.

11. *Fun* (The surprise exam paradox) Your class meets 5 days a week, and on Friday your teacher tells you that there will be a surprise exam next week, meaning (1) that there will be an exam, and (2) that it will be a surprise (i.e., you won't be able to anticipate the night before the exam that the exam will be the next day). What can you conclude about the exam? Relate this problem to the Centipede Game discussed previously.

    Well, the exam can't be on Friday, because then on Thursday night you'd think, "Aha! The exam's got to be Friday!" So then you wouldn't be surprised; so the exam can't be on Friday. But then the exam can't be on Thursday, because then on Wednesday night you'd think, "Aha! The exam can't be on Friday, so it's got to be Thursday!" So then you wouldn't be surprised; so the exam can't be on Thursday. But then the exam can't be on Wednesday, or Tuesday, or even Monday. An apparently noncontroversial statement by your teacher turns out to be quite treacherous!

Chapter 9: Simultaneous Move Games

1. Everybody in City X drives to work, so commutes take two hours. Imagine that a really good bus system could get everybody to work in 40 minutes if there were no cars on the road. There are only two hitches: (1) If there are cars on the road, the bus gets stuck in traffic just like every other vehicle, and therefore (2) people can always get to their destination 20

minutes faster by driving instead of taking the bus (the extra 20 minutes comes from walking to the bus stop, waiting for the bus, etc.).

(a) If such a bus system were adopted in City X and each resident of City X cared only about getting to work as quickly as possible, what would you expect the outcome to be?

A good prediction is that everybody would drive to work because driving is a dominant strategy: no matter what everybody else does, you always get there 20 minutes faster by driving.

(b) Is this outcome Pareto efficient? Explain briefly.

This outcome is not Pareto efficient because the commute takes 2 hours; a Pareto improvement would be for everybody to take the bus, in which case the commute would only take 40 minutes.

(c) "The central difficulty here is that each commuter must decide what to do without knowing what the other commuters are doing. If you knew what the others decided, you would behave differently." Do you agree with this argument?

The central difficulty is *not* that you don't know what others are going to do; you have a dominant strategy, so the other players' strategies are irrelevant for determining your optimal strategy.

(d) What sort of mechanism do you suggest for reaching the optimal outcome in this game? Hint: Make sure to think about enforcement!

A reasonable mechanism might be passing a law that everybody has to take the bus or pay a large fine.

2. (The Public/Private Investment Game) You are one of ten students in a room, and all of you are greedy income-maximizers. Each student has \$1 and must choose (without communicating with the others) whether to invest it in a private investment X or a public investment Y. Each dollar invested in the private investment X has a return of \$2, which goes entirely to the investor. Each dollar invested publicly has a return of \$10, which is divided equally among the ten students (even those who invest privately). So if six students invest publicly, the total public return is \$60, divided equally among the ten students; the four students who invested privately get an additional \$2 each from their private investment.

(a) What outcome do you predict in the simultaneous-move game, i.e., if all the students must write down their investment decisions at the same time?

A good prediction is that everybody will invest in the private good because it's a dominant strategy: no matter what everybody else does, you always get \$1 more by investing privately.

(b) Is this outcome Pareto efficient? If not, identify a Pareto improvement.

This outcome is not Pareto efficient because each player only gets a return of \$2; a Pareto improvement would be for everybody to invest in the public good, in which case each player would get a return of \$10.

(c) "The central difficulty here is that the students must decide without knowing what the other students are doing. If you knew what the other students decided, you would behave differently." Do you agree with this argument?

The central difficulty is *not* that you don't know what others are going to do; you have a dominant strategy, so the other players' strategies are irrelevant for determining your optimal strategy.

(d) If communication were possible, what sort of mechanism do you suggest for reaching the optimal outcome in this game? Hint: Make sure to think about enforcement!

A reasonable mechanism might be passing a law that everybody has to invest in the public good or pay a large fine.

3. Consider the following game.

		Player 2	
		D	C
Player 1	D	$-2, -2$	$10, -5$
	C	$-5, 10$	$1, 1$

(a) What outcome do you expect if this game is played once? Explain briefly.

Playing D is a dominant strategy for each player, so we can expect an outcome of (D, D).

(b) (5 points) What outcome do you expect if this game is played twice? Explain briefly.

Using backward induction, we start at the end of the game, i.e., in the second round. Playing D is a dominant strategy for each player in this round, so we can expect an outcome of (D, D) in the second round. But the players will anticipate this outcome, so playing D becomes a dominant strategy in the first round too! As a result, the expected outcome is for both players to play D both times.

Chapter 10: Application: Auctions

1. *Fun/Challenge* The website freemarkets.com runs procurement auctions: companies in need of supplies post information about their purchasing

needs (e.g., so and so many sheets of such and such kind of glass) and the maximum amount they're willing to pay for those purchases; bidders then bid the price down, and the lowest bidder receives that price for the specified products. The ads for freemarkets.com say things like, "At 1pm, Company X posted a request for 1 million springs, and indicated that it was willing to pay up to \$500,000. By 4pm, the price was down to \$350,000."

(a) Explain how an auction can help Company X get a low price on springs.

Auctions pit different suppliers against each other, and their individual incentives lead them to drive down the price. This helps ensure that Company X will not be paying much more for springs than it costs the suppliers to produce them.

(b) Is the example ad above impressive? Is it susceptible to gaming (i.e., strategic manipulation)?

The example in the ad above may not be as impressive as it sounds because of the potential for gaming: if Company X knows that a number of firms can produce the springs for about \$350,000, it has essentially nothing to lose by indicating a willingness-to-pay of \$500,000—or even \$1,000,000—because the auction dynamics will drive the price down toward \$350,000. An analogy may help: say I want to purchase a \$20 bill. As long as there are enough competitive bidders, I can more-or-less fearlessly say that I'm willing to pay up to \$1,000 for that \$20 bill; competitive pressures will force the winning bid down to about \$20.

2. You're a bidder in a second-price sealed-bid auction. Your task here is to explain (as if to a mathematically literate non-economist) why you should bid your true value.

(a) Explain (as if to a non-economist) why you cannot gain by bidding *less* than your true value.

The intuition can be seen from an example: say you're willing to pay up to \$100, but you only bid \$90. Let y be the highest bid not including your bid. If $y < 90$ then you win the auction and pay y; in this case, bidding \$90 instead of \$100 doesn't help you or hurt you. If $y > 100$ then you lose the auction and would have lost even if you bid \$100; again, bidding \$90 instead of \$100 doesn't help you or hurt you. But if y is between \$90 and \$100 (say, $y = \$95$) then bidding \$90 instead of \$100 actively hurts you: you end up losing the auction when you would have liked to have won it. (You had a chance to get something you value at \$100 for a payment of only \$95, but you didn't take it.)

(b) Explain (as if to a non-economist) why you cannot gain by bidding *more* than your true value.

Again, the intuition can be seen in the same example in which you're willing to pay up to \$100. Assume that you bid \$110 and that y is the highest bid not including your bid. If $y <$ \$100 then you win the auction and pay y; in this case bidding \$110 instead of \$100 doesn't help you or hurt you. If $y >$ \$110 then you lose the auction; again, bidding \$110 instead of \$100 doesn't help you or hurt you. But if y is between \$100 and \$110 (say, $y =$ \$105) then bidding \$110 instead of \$100 actively hurts you: you end up winning the auction when you would have liked to have lost it. (You pay \$105 for something you only value at \$100.)

3. You're a bidder in a first-price sealed bid auction. Should you bid your true value, more than your true value, or less than your true value? Explain briefly, as if to a mathematically literate non-economist.

   You should bid less than your true value. If your true way is, say, \$100, then you are indifferent between having the object and having \$100. If you bid \$100, winning the auction won't make you better off; if you bid more than \$100, winning the auction will actually make you worse off. The only strategy that makes it possible for you to be better off is for you to bid less than \$100.

4. Your mathematically literate but non-economist friend Jane owns one of the few original copies of *Send This Jerk the Bedbug Letter!*, a best-selling book about playing games with giant corporations. She decides to auction off the book to raise money for her new dot.com venture. She tells you that she's going to use a first-price sealed bid auction. You ask her why she doesn't use a second-price sealed bid auction , and she looks at you like you're nuts: "Look, dummy, I'm trying to make as much money as I can. Why would I charge the second-highest bid price when I can charge the highest bid price?!?" Write a response.

   A reasonable response might start off by noting that bidders will behave differently in the two auctions: bidders will shade their bids in a first-price auction, but not in a second-price auction. So in a first-price auction you get the highest bid from among a set of relatively low bids, and in a second-price auction you get the second-highest bid from among a set of relatively high bids. It's no longer clear which auction has the higher payoff. (In fact, there is a deeper result in game theory, called the Revenue Equivalence Theorem, which predicts that both types of auctions will yield the same expected payoff.)

5. We can use the expected value calculations from Chapter 2 to get another perspective on bidding in first- and second-price sealed bid auctions.

   (a) The first step in calculating expected values is determining the different possible outcomes. So: what are the possible outcomes of bidding $\$x$ in an auction?

There are two possible outcomes: either $\$x$ is the highest bid and you win the auction, or $\$x$ isn't the highest bid and you lose the auction.

(b) Next: write down and simplify an expression for the expected value of bidding $\$x$ in an auction. Use Value(Winning) to denote the value of winning the auction. Assume that the value of losing the auction is zero.

Your expected value from bidding $\$x$ in the auction is

$$\begin{aligned} \text{EV(Bidding } \$x) &= \text{Pr(Your } \$x \text{ bid wins)} \cdot \text{Value(Winning)} \\ &\quad + \text{Pr(Your } \$x \text{ bid loses)} \cdot \text{Value(Losing)}. \end{aligned}$$

Since the value of losing is zero (you get nothing, you pay nothing), the second term disappears. So your expected value boils down to something like

$$\text{EV(Bidding } \$x) = \text{Pr(Your } \$x \text{ bid wins)} \cdot \text{Value(Winning)}.$$

(c) Write down an expression for the expected value of bidding $\$x$ in a first-price sealed bid auction. Assume that your gain or "profit" from winning an auction is the difference between your true value for the item and the price you actually have to pay for it. Can you use this expected value expression to highlight the issues faced by a bidder in such an auction? For example, can you show mathematically why bidders should shade their bids?

The expression above simplifies to

$$\text{EV(Bidding } \$x) = \text{Pr(Your } \$x \text{ bid wins)} \cdot (\text{Value of object} - \$x).$$

Here we can see that bidding your true value is a bad idea: your expected value will never be greater than zero! We can also see the tension at work in first-price sealed bid auctions: by reducing your bid, you lower the probability that you will win, but you increase the value of winning. (Optimal bidding strategies in this case are complicated. How much to shade your bid is a difficult question, since it depends on how much you think other people will bid....)

(d) Write down an expression for the expected value of bidding $\$x$ in a second-price sealed bid auction. (Again, assume that your gain or "profit" from winning an auction is the difference between your true value for the item and the price you actually have to pay for it.) Can you use this expected value expression to highlight the issues faced by a bidder in such an auction? For example, can you show mathematically why bidders should bid their true value?

Your expected value of bidding $\$x$ reduces to

$$\text{EV(Bidding \$x)} = \text{Pr(Your } \$x \text{ bid wins)} \cdot (\text{Value of object} - \$y)$$

where $\$y$ is the *second-highest* bid. Since the price you pay is not determined by your own bid, shading your bid below your true value doesn't help you. It only increases the probability that you will lose the bid when you would like to have won it. (The same is true for bidding over your true value. This only increases the probability that you will win the object and be forced to pay an amount greater than your true value.) You maximize your expected value by bidding your true value.

Chapter 11: Application: Marine Affairs

There are no problems in this chapter.

Chapter 12: Transition: Game Theory v. Price Theory

1. Would you say that that market for new cars is a competitive market? Why or why not? How about the market for used cars?

   Ford, GM, Toyota, and a few other manufacturers dominate the market for new cars, so it is not a competitive market. In contrast, the market for used cars is pretty close to the competitive ideal. There are lots of small sellers—individuals looking to sell their cars—and lots of small buyers—individuals looking to buy a used car.

Answer Key for Part III: Many v. Many

Chapter 13: Supply and Demand: The Basics

1. Explain, as if to a non-economist, why the intersection of the market supply curve and the market demand curve identifies the market equilibrium.

   This is the price at which the amount that buyers want to buy equals the amount that sellers want to sell. At a higher price, sellers want to sell more than buyers want to buy, creating incentives that push prices down toward the equilibrium. At a lower price, buyers want to buy more than sellers want to sell, creating incentives that push prices up toward the equilibrium.

2. For each item, indicate the likely impact on the supply and demand for wine. Then indicate the effect on the equilibrium price and quantity. It may help to use a graph.

   (a) The legal drinking age for wine is lowered to 18.

   (b) A fungus destroys part of the grape harvest. (Grapes are **inputs** in wine-making, as are labor, machinery, and glass.)

   (c) The price of cheese increases. (Wine and cheese are **complements** or **complementary goods**, as are skis and ski boots; monitors and keyboards; and peanut butter and jelly.)

   (d) The price of beer falls. (Beer and wine are **substitutes**, as are eyeglasses and contact lenses; burritos and hamburgers; and pens and pencils.)

   (a) Demand increases. Equilibrium price up, quantity up.

   (b) Supply decreases. Equilibrium price up, quantity down.

   (c) Demand decreases. Equilibrium price down, quantity down.

   (d) Demand decreases. Equilibrium price down, quantity down.

3. For each item, indicate the likely impact on the supply and demand for popsicles in Hawaii. Then indicate the effect on the equilibrium price and quantity. It may help to use a graph.

   (a) More tourists visit Hawaii.
   (b) An arsonist burns down half of the popsicle factories in Hawaii.

   (a) Demand increases. Equilibrium price up, quantity up.
   (b) Supply decreases. Equilibrium price up, quantity down.

4. For each item, indicate the likely impact on the supply and demand for codfish. Then indicate the effect on the equilibrium price and quantity. It may help to use a graph.

   (a) News reports that cod contains lots of omega-3 fatty acids, which are great for your health.
   (b) Overfishing drastically reduce the fish population.

   (a) Demand increases. Equilibrium price up, quantity up.
   (b) Supply decreases. Equilibrium price up, quantity down.

5. For each item, indicate the likely impact on the supply and demand for paperback books. Then indicate the effect on the equilibrium price and quantity. It may help to use a graph.

   (a) The invention (and widespread use) of the printing press.
   (b) The invention (and widespread use) of the television.
   (c) The invention (and widespread use) of "book lights" (the small clip-on lights that allow people to read at night without disturbing their spouses/partners/etc.)
   (d) News reports that reading books is a cure for stress and high blood pressure.
   (e) A decrease in the price of paper.

   (a) Supply increases. Equilibrium price down, quantity up.
   (b) Demand decreases. Equilibrium price down, quantity down.
   (c) Demand increases. Equilibrium price up, quantity up.
   (d) Demand increases. Equilibrium price up, quantity up.
   (e) Supply increases. Equilibrium price down, quantity up.

6. For each item, indicate the likely impact on the supply and demand for bulldozer operators and other skilled construction workers. (It may help to think for a moment about who the suppliers and demanders are for these services.) Then indicate the effect on the equilibrium price and quantity. It may help to use a graph.

(a) The elimination of vocational programs that teach people how to use bulldozers.

(b) A huge increase in the number of well-paying service-sector jobs such as computer programming.

(c) A fall in the price of bulldozers and other construction equipment. (To state the obvious: bulldozers and bulldozer operators are complements, like bread and butter or computers and monitors.)

(d) An increase in the wage for unskilled laborers. (To state the less obvious: skilled labor (e.g., workers who can use bulldozers) and unskilled labor (e.g., workers who can only use shovels) are substitutes, as are tea and coffee and planes, trains, and automobiles.)

(a) Supply decreases. Equilibrium price up, quantity down.

(b) Supply decreases. Equilibrium price up, quantity down.

(c) Demand increases. Equilibrium price up, quantity up.

(d) Demand increases. Equilibrium price up, quantity up.

7. A few quarters ago a libertarian named Joel Grus came to class, and one of the things he discussed was drug legalization. So: for each item below, indicate the likely impact on the supply and demand for cocaine. Then indicate the effect on the equilibrium price and quantity. It may help to use a graph.

(a) Military aid to Columbia to help that country exterminate the coca plant from which cocaine is made.

(b) Sentences of life in prison for cocaine traffickers and members of drug-selling gangs.

(c) Sentences of life in prison for buyers of cocaine.

(d) Drug treatment programs to try to help addicts stop using drugs.

(e) News reports that eating nutmeg has the same effect as snorting cocaine.[3]

(f) Finally: Imagine (hypothetically) that legalizing cocaine would have no effect on the demand for that drug. Describe the impact of legalization on the market for cocaine.

(g) This question is a follow-up to the previous one: Would the legalization of cocaine necessarily lead to more money being spent on cocaine, i.e., would total expenditures necessarily go up?

(a) Supply decreases. Equilibrium price up, quantity down.

[3]It doesn't, really. But nutmeg *is* a hallucinogen, if taken by the tablespoonful. Based on my experience as a camp counselor, I'd recommend against it: you're almost sure to end up in the hospital, and maybe in the morgue. . . .

(b) Supply decreases. Equilibrium price up, quantity down.

(c) Demand decreases. Equilibrium price down, quantity down.

(d) Demand decreases. Equilibrium price down, quantity down.

(e) Demand decreases. Equilibrium price down, quantity down.

(f) Supply increases. Equilibrium price down, quantity up.

(g) Not necessarily. The equilibrium quantity would go up, but the equilibrium price would go down. The net impact on total expenditures pq is therefore unclear.

8. For each item, indicate the likely impact on the supply and demand for beef. Then indicate the effect on the equilibrium price and quantity. It may help to use a graph.

   (a) Mad cow disease scares away meat-eating shoppers.

   (b) All the cows in England are slaughtered and thrown into the ocean.

   (c) New drugs make cows grow faster at lower cost to farmers.

   (d) The price of chicken falls.

   (e) News reports that eating red meat is bad for you.

   (a) Demand decreases. Equilibrium price down, quantity down.

   (b) Supply decreases. Equilibrium price up, quantity down.

   (c) Supply increases. Equilibrium price down, quantity up.

   (d) Demand decreases. Equilibrium price down, quantity down.

   (e) Demand decreases. Equilibrium price down, quantity down.

9. Read the following excerpt from the *New York Times* of October 27, 2000.

> For 10 years, British officials consistently misled the public by deliberately playing down the possibility that mad-cow disease could be transmitted to humans, an official report said today. . . The 4,000-page report, published after a three-year investigation. . . severely criticized the "culture of secrecy" that characterized the government's response to a crisis that has wreaked havoc with Britain's once-proud beef industry, forced the slaughter of almost four million cows and led to the deaths so far of 77 Britons. . . "My own personal belief would be that we are more likely looking in the region of a few hundred to several thousand more" victims, Prof. Peter Smith, acting head of the government's advisory committee on the disease, said on television this morning. "But it must be said that we can't rule out tens of thousands."

> [It was] "a consuming fear of provoking an irrational public scare"... that caused a government veterinary pathologist to label "confidential" his first memo on mad-cow disease in 1986; that led John Gummer, then the agriculture minister, to make a show of publicly feeding a hamburger to his 4-year-old daughter, Cordelia, in 1990; and that led Britain's chief medical officer in 1996 to declare, "I myself will continue to eat beef as part of a varied and balanced diet." ... At the same time, government policy was marred by bureaucratic bungling, a lack of coordination between departments and the fact that the Ministry of Agriculture, Fisheries and Food had two somewhat contradictory missions: to protect consumers and to support the beef industry.

(a) What was the effect of mad-cow disease on the demand curve for British beef? Draw a supply and demand graph and indicate the effect on the equilibrium.

(b) What was the effect of mad-cow disease on the supply curve for British beef? Draw a supply and demand graph and indicate the effect on the equilibrium.

(c) Combining your answers to the previous two questions, can you predict with certainty what happened to the equilibrium price for British beef? What about the equilibrium quantity?

(d) The British government put together quite a PR campaign in their effect to avoid "an irrational public scare". Would such a public scare have pleased or displeased the following groups?

   i. Die-hard beef eaters
   ii. Die-hard chicken eaters
   iii. Beef suppliers
   iv. Chicken suppliers

(a) Demand decreases. Equilibrium price down, quantity down.

(b) Supply decreases. Equilibrium price up, quantity down.

(c) Equilibrium quantity fell. The effect on equilibrium price is unclear, since the demand shift and the supply shift move in opposite directions.

(d) A public scare would decrease demand for beef and increase demand for chicken. The equilibrium price and quantity of beef would therefore fall, and the equilibrium price and quantity of chicken would rise. These changes would benefit beef eaters and chicken suppliers and hurt chicken eaters and beef suppliers.

10. Read the following excerpt from the *New York Times* of October 5, 2000.

> The energy proposals that Mr. Bush, the Republican presidential candidate, brought out last week—including opening part of the Arctic National Wildlife Refuge to exploration and incentives to promote coal and nuclear power—could test the willingness of Americans to rebalance environmental and energy priorities in the face of higher prices. For his part, Vice President Al Gore, the Democratic presidential candidate, favors investments in mass transit and incentives to encourage the use of more fuel-efficient vehicles and alternative energy sources.

The "energy crisis" was a big topic in the presidential race. (It might be interesting to investigate how the real price of gasoline has changed over the last 30 or so years.) For each item, indicate the likely impact on the supply and demand for oil. Then indicate the effect on the equilibrium price and quantity. It might help to use a graph. Please note that, in addition to being refined to make gasoline for cars, oil is also used to heat homes and to produce electricity; coal and nuclear power are also used to produce electricity.

(a) Opening part of the Arctic National Wildlife Refuge to oil exploration.

(b) Government incentives to promote coal and nuclear power.

(c) Government investments in mass transit.

(d) Government incentives to encourage the use of solar-powered vehicles.

(e) Will all of these policies reduce the price of oil? Yes No (Circle one)

(f) Will all of these policies reduce the consumption of oil? Yes No (Circle one)

(g) Is it correct that Bush's proposals all address the supply side of the problem?

(h) Is it correct that Gore's proposals all address the demand side of the problem?

(a) Supply increases. Equilibrium price down, quantity up.

(b) Demand decreases. Equilibrium price down, quantity down.

(c) Demand decreases. Equilibrium price down, quantity down.

(d) Demand decreases. Equilibrium price down, quantity down.

(e) Yes.

(f) No. Opening the Arctic National Wildlife Refuge would increase the equilibrium quantity.

(g) No. Promoting substititutes to oil (e.g., coal and nuclear power) is a demand-side strategy.

(h) Yes.

11. Let's look a little more closely at one of now-President Bush's energy proposals: opening up the Arctic National Wildlife Refuge (ANWR) to oil drilling.

    When you answered the previous question, you probably assumed that that oil would become available immediately, i.e., that oil companies could immediately begin extracting and selling that oil. (I wanted you to assume that, so do not go back and rethink your answer above!) It turns out that life is more complicated than that: it takes time to build pipelines and to drill oil wells into pristine Artic wilderness, so any oil that comes from ANWR will not reach the market for something like 5 years. This fact became a source of contention during the presidential campaign, with Al Gore arguing that opening ANWR would have no effect on current gasoline prices because of this 5-year time lag, and George W. Bush arguing. . . well, I don't remember what his argument was, but it probably had something to do with fuzzy math and how when people take actions there have to be consequences.

    Unlike the majority of the American public, you now understand how supply and demand works, and you should be able to assess the validity of Al's argument. You should try to do this on your own; otherwise (or once you try it on your own), the questions below can serve to guide and/or confirm your thinking.

    (a) Think ahead five years into the future (to 2006), when that oil from ANWR will finally reach the market. Indicate the effect this will have on the market for oil five years from now. (You should draw a supply and demand graph.)
    (b) Next: looking at your graph, what is the effect on the market price for oil in 2006? Will it be higher, lower, or the same?
    (c) Next: Come back to the year 2001. We need to figure out the impact of that future price change on the market for oil today. So: imagine that you own a bunch of oil. You're trying to decide whether to invest in the bank (by extracting and selling the oil and putting the money in the bank) or to "invest in the oil" (by leaving the oil in the ground until, say, 2006). Does your answer to the previous question make investing in oil look more attractive or less attractive?
    (d) Next: As a result, are you likely to sell more oil this year or less oil?
    (e) Finally, think about what this means in terms of your individual supply curve, and remember that all the oil companies are thinking just like you. So: use a supply and demand graph to determine the effect on oil prices today of opening up ANWR for oil drilling. Does today's price go up, down, or stay the same?

    (a) Supply increases. Equilibrium price down, quantity up.
    (b) Lower.

(c) Less attractive.

(d) More oil.

(e) Supply increases. Equilibrium price down, quantity up.

Chapter 14: Taxes

1. Explain, as if to a mathematically literate non-economist, why taxes shift the supply and/or demand curves the way they do. (The same answer works for sales taxes and per unit taxes, as well as for taxes on buyers and taxes on sellers.)

   This is the logic identified in the text, e.g., "At a price of x with a \$.40 tax, buyers should be willing to buy exactly as much as they were willing to buy at a price of $\$(x + .40)$ without the tax." For per-unit taxes, you can also use the ideas of marginal cost and marginal benefit: a tax on the sellers increases marginal costs by the amount of the tax, and a tax on the buyers reduces marginal benefits by the amount of the tax. (Applying this marginal approach is a bit tricky for ad valorem taxes such as sales taxes. You need an additional assumption here about firm profits in equilibrium. . .)

2. Figure A.4 shows a hypothetical market for oranges. Use it (and the replicas on the following pages) to answer the questions in the remaining problems in this chapter.

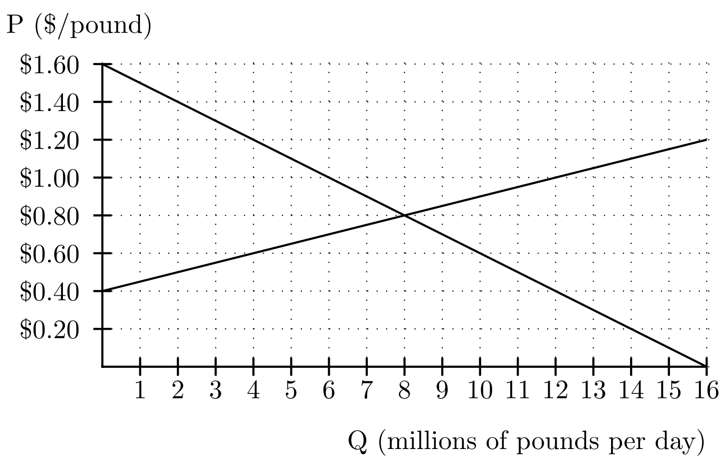


Figure A.4: A hypothetical market for oranges

(a) What is the equilibrium price and quantity? (Use correct units!)

(b) Calculate the slope of the supply curve and the slope of the demand curve. (Recall that slope is rise over run, e.g., $S_D = \frac{\Delta p}{\Delta q}$.) Calculate the ratio of the slopes $\left(\frac{S_D}{S_S}\right)$.

(a) The equilibrium price is \$.80 per pound; the equilibrium quantity is 8 million pounds per day.

(b) To find the slope of the supply curve, pick any two points on it—say, $(8, .80)$ and $(12, 1.00)$. Then the slope of the supply curve is

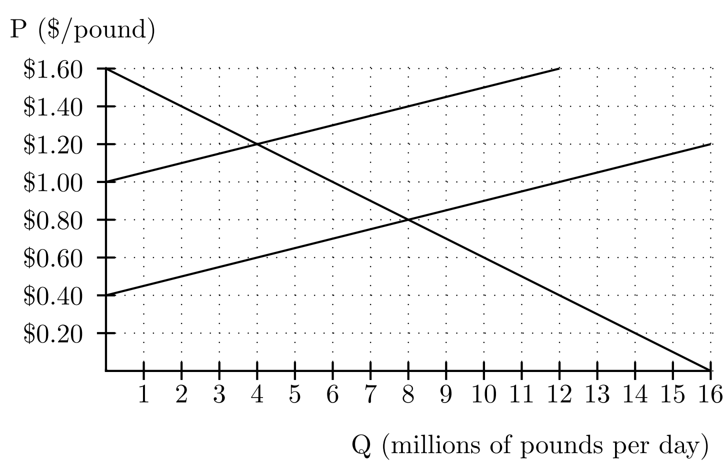
$$S_S = \frac{\text{rise}}{\text{run}} = \frac{1.00 - .80}{12 - 8} = \frac{.20}{4} = .05.$$

Similarly, to find the slope of the demand curve, pick any two points on it—say $(8, .80)$ and $(12, .40)$. Then the slope of the demand curve is

$$S_D = \frac{\text{rise}}{\text{run}} = \frac{.40 - .80}{12 - 8} = \frac{-.40}{4} = -.1.$$

So the ratio of the two slopes is $\left(\frac{S_D}{S_S}\right) = \frac{-.1}{.05} = -2.$

3. Suppose that the government imposes an excise tax of \$.60 per pound on the sellers of oranges.



(a) Show the impact of this tax on the supply and demand curves.

(b) At the new equilibrium, how many oranges will people eat? (Use correct units!)

(c) Calculate the total tax revenue for the government from this tax. (Use correct units!)

(d) How much do the buyers pay for each pound of oranges?

(e) How much after-tax revenue do the sellers receive for each pound of oranges?

(f) Use your answers to the previous two questions to determine the economic incidence of the tax. In other words, calculate the amount of the tax burden borne by the buyers (T_B) and by the sellers (T_S), and the ratio $\dfrac{T_B}{T_S}$.

(g) Compare the tax burden ratio with the ratio of the slopes from problem 2b. Can you explain the intuition behind this result?

(a) The supply curve shifts up by \$.60.

(b) The new equilibrium features a price of \$1.20 per pound and a quantity of 4 million pounds per day.

(c) A tax of \$.60 per pound levied on 4 million pounds per day yields revenue of (\$.60)(4) = \$2.4 million per day.

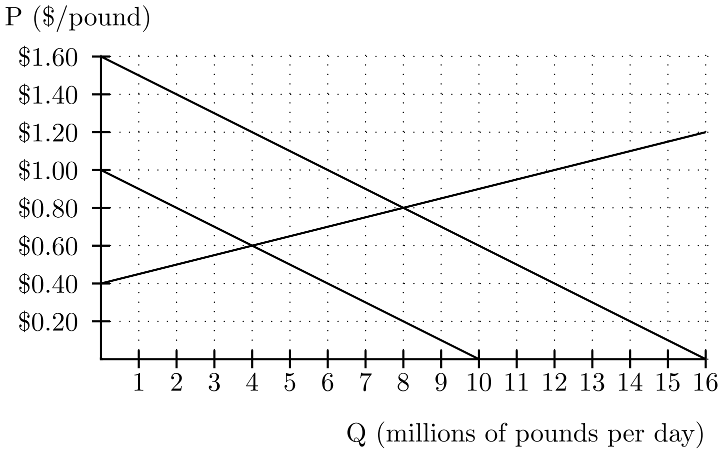
(d) The buyers pay \$1.20 for each pound of oranges.

(e) Before paying the \$.60 tax the sellers receive \$1.20 per pound, so after paying the tax the sellers receive \$.60 per pound.

(f) Without the tax, buyers paid \$.80 per pound; they now pay \$1.20 per pound, so they are worse off by $T_B = \$.40$ per pound. Similarly, the sellers received \$.80 per pound without the tax, but now they only receive \$.60 per pound, so they are worse off by $T_S = \$.20$ per pound. (As a check here, note that the \$.40 per pound impact on the buyers plus the \$.20 per pound impact on the sellers equals the \$.60 per pound tax.) The ratio of the tax burdens is $\dfrac{T_B}{T_S} = \dfrac{.40}{.20} = 2$.

(g) The tax burden ratio is the same magnitude as the ratio of the slopes calculated previously! Intuitively, this is because the ratio of the slopes measures the relative responsiveness of buyers and sellers to price changes. The side that is most responsive to price changes (in this case, the sellers) can push the lion's share of the tax burden onto the other side.

4. Answer the same questions as in problem 3, but now suppose that the government imposes a per-unit tax of \$.60 per pound on the buyers of oranges. (Recall that the buyer now has to pay the government *in addition to* paying the seller.)



(a) The demand curve shifts down by \$.60.

(b) The new equilibrium features a price of \$.60 per pound and a quantity of 4 million pounds per day.

(c) A tax of \$.60 per pound levied on 4 million pounds per day yields revenue of (\$.60)(4) = \$2.4 million per day.

(d) The buyers pay \$.60 to the sellers for each pound of oranges, plus the tax of \$.60 to the government, so they pay a total of \$1.20 per pound.

(e) The sellers receive \$.60 per pound of oranges.

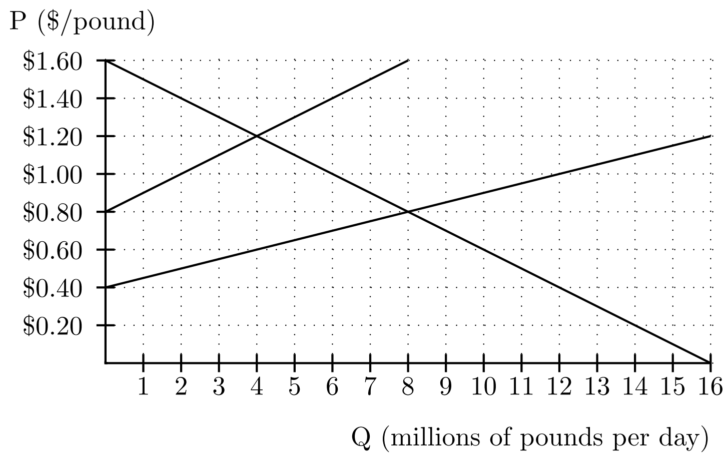
(f) Without the tax, buyers paid \$.80 per pound; they now pay \$1.20 per pound, so they are worse off by $T_B = \$.40$ per pound. Similarly, the sellers received \$.80 per pound without the tax, but now they only receive \$.60 per pound, so they are worse off by $T_S = \$.20$ per pound. (As a check here, note that the \$.40 per pound impact on the buyers plus the \$.20 per pound impact on the sellers equals the \$.60 per pound tax.) The ratio of the tax burdens is $\frac{T_B}{T_S} = \frac{.40}{.20} = 2$.

(g) The tax burden ratio is the same magnitude as the ratio of the slopes calculated previously!

5. How do your answers in problem 4 compare with those in problem 3? What does this suggest about the difference between a per-unit tax on buyers and a per-unit tax on sellers?

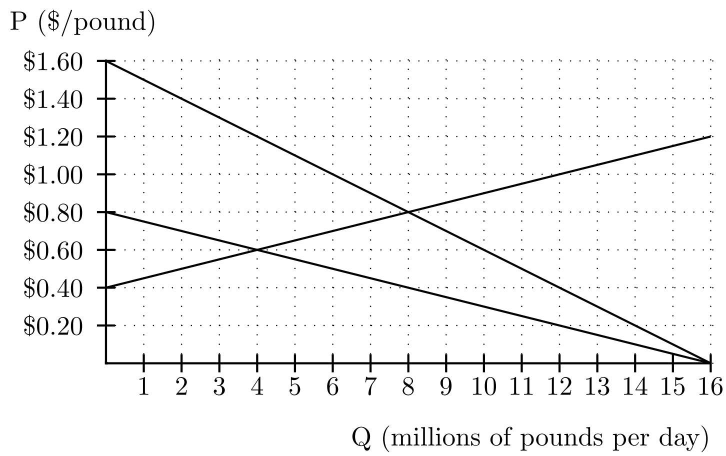
   The answers are essentially identical—regardless of the *legal* incidence of the tax (i.e., whether it's levied on the buyers or the sellers), the *economic* incidence of the tax comes out the same, i.e., the buyers always end up bearing \$.40 of the tax burden and the sellers end up bearing \$.20 of the tax burden. This is an example of the **tax incidence** result.

6. Answer the same questions as in problem 3, but now suppose that the government imposes a sales tax of 50% on the sellers of oranges. (With a sales tax, if sellers sell a pound of oranges for \$1, they get to keep \$.50 and have to pay the government \$.50; if they sell a pound of oranges for \$2, they get to keep \$1 and have to pay the government \$1.)



The supply curve rotates as shown above. The remaining answers are identical to those above; note that at the equilibrium price of \$1.20 per pound, the 50% tax on the sellers amounts to \$.60 per pound.

7. Answer the same questions as in problem 3, but now suppose that the government imposes a sales tax of 100% on the buyers of oranges. (If buyers buy a pound of oranges for \$1, they have to pay the seller \$1 and the government \$1; if they buy a pound of oranges for \$2, they have to pay the seller \$2 and the government \$2.)



The demand curve rotates as shown above. The remaining answers are identical to those above; note that at the equilibrium price of $.60 per pound, the 100% tax on the buyers amounts to $.60 per pound.

8. How do your answers in problem 7 compare with those in problem 6? What does this suggest about the difference between a sales tax on buyers and a sales tax on sellers?

   The answers are the same! Again, this is an example of tax equivalence, but with sales taxes the comparison is a little bit less obvious than with per-unit taxes: in the case of sales taxes, a 50% tax on the sellers is equivalent to a 100% tax on the buyers. This is because the market price is the price the buyer pays the seller; since the market price is twice as high when the tax is on the seller ($1.20 versus $.60), the sales tax rate on the sellers needs to be only half as high as the sales tax rate on the buyers in order to yield an equivalent result.

Chapter 15: Elasticities

1. Go over previous problems (and/or make up some of your own) and calculate the elasticities of demand and supply at various points.

2. Let's compare short-run elasticities and long-run elasticities.

   (a) Draw a demand curve. Label the axes (p and q) and label this curve "SR". Pick a point on your demand curve (any point!). Assume that the curve you have drawn is a short-run demand curve, and that the point you have drawn is the short-run equilibrium. Now, assume that the market equilibrium in the long run is at the same point. Will the long-run demand curve be flatter or steeper than the short-run curve?

Draw in what the long-run demand curve might look like, labeling it "LR". Explain. (Hint: Think about whether buyers' responsiveness to price changes is likely to increase or decrease over time.)

(b) Draw a supply curve. Label the axes (p and q) and label this curve "SR". Pick a point on your supply curve (any point!). Assume that the curve you have drawn is a short-run supply curve, and that the point you have drawn is the short-run equilibrium. Now, assume that the market equilibrium in the long run is at the same point. Will the long-run supply curve be flatter or steeper than the short-run curve? Draw in what the long-run supply curve might look like, labeling it "LR". Explain. (Hint: Think about whether sellers' responsiveness to price changes is likely to increase or decrease over time.)

Long-run demand and supply curves are flatter, i.e., more elastic. This is because responsiveness to price changes increases in the long run. For example, in the long run sellers can build extra factories or close down existing factories when their useful lifetime expires. Similarly, in the long run buyers of gasoline can respond to a price increase by buying a different car or moving where they live and/or where they work. Short-run options are more limited, leading to reduced responsiveness to price changes.

3. Figure A.5 shows a hypothetical demand curve for oranges.

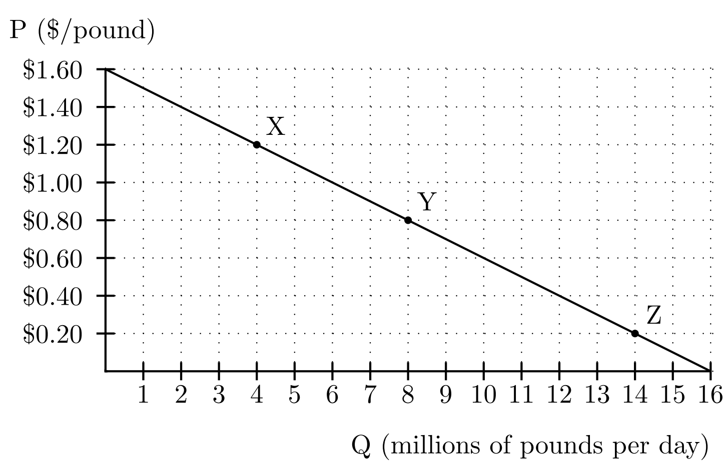


Figure A.5: A hypothetical market for oranges

(a) Calculate the price elasticity of demand at point Y.

(b) During normal years, the supply curve is such that point Y is the equilibrium. Of the other two points, one is the equilibrium during "bad" years (when frost damages the orange crop), and one is the equilibrium during "good" years (when the orange crop thrives). Which one is point X?

(c) What is the total revenue at points X, Y, and Z? (Use correct units!)

(d) The orange growers' profit is total revenue minus total costs. If total costs are the same in all years, do the growers have higher profits in "bad" years or "good" years? Can you explain what's going on here?

(e) This demand curve is the same as in problems 2–8 in Chapter 14. Go back to those problems, calculate the price elasticity of supply at the original equilibrium, and combine your answer there with your answer from problem 3a above to calculate the ratio of the elasticities $\left(\dfrac{\epsilon_S}{\epsilon_D}\right)$. Compare the results with the tax burden ratios and the slope ratios you calculated in those previous problems.

(a) The point Y corresponds to point A in the elasticity formula, so we have $p_A = \$.80$ and $q_A = 8$. For point B we can take any other point, e.g., the convenient point with $p_B = \$.60$ and $q_B = 10$. Then

$$\varepsilon = \frac{q_B - q_A}{p_B - p_A} \cdot \frac{p_A}{q_A} = \frac{2}{-.20} \cdot \frac{.80}{8} = -10 \cdot \frac{1}{10} = -1.$$

(b) Point X is the equilibrium during bad years, when frost reduces supply.

(c) Total revenue at the three points are pq, i.e., $(1.2)(4) = \$4.8$ million per day at point X, $(.8)(8) = \$6.4$ million per day at point Y, and $(.2)(14) = \$2.8$ million per day at point Z.

(d) Growers make higher profits during "bad" years: their revenue is higher and their costs are assumed to be identical. This is basically a Prisoner's Dilemma situation for the growers: they would all be better off if they could restrict supply during "good" years, but the individual incentives lead them to flood the market and get low prices and low profits.

(e) The point Y corresponds to point A in the elasticity formula, so we have $p_A = \$.80$ and $q_A = 8$. For point B we can take any other point on the supply curve, e.g., the convenient point with $p_B = \$.60$ and $q_B = 4$. Then

$$\varepsilon = \frac{q_B - q_A}{p_B - p_A} \cdot \frac{p_A}{q_A} = \frac{-4}{-.20} \cdot \frac{.80}{8} = 20 \cdot \frac{1}{10} = 2.$$

So the ratio of the elasticities is $\dfrac{\epsilon_S}{\epsilon_D} = \dfrac{2}{-1} = -2$. This is the same as the ratio of the slopes calculated previously! (This result follows from problem 4.)

4. Show mathematically that the ratio of the elasticities of supply and demand is the inverse of the ratio of the slopes of the supply and demand curves, i.e., $\left(\frac{\epsilon_S}{\epsilon_D}\right) = \left(\frac{S_D}{S_S}\right)$.

$$\frac{\varepsilon_S}{\varepsilon_D} = \frac{\dfrac{\Delta q_S}{\Delta p_S}\dfrac{p}{q}}{\dfrac{\Delta q_D}{\Delta p_D}\dfrac{p}{q}} = \frac{\dfrac{\Delta p_D}{\Delta q_D}}{\dfrac{\Delta p_S}{\Delta q_S}} = \frac{S_D}{S_S}.$$

5. Very long run supply curves are often assumed to be perfectly elastic.

   (a) Explain the intuition behind perfectly elastic long run supply curves. (Hint: Recall that comparable assets should have comparable rates of return. Think about whether or not a random firm will decide to start producing widgets.)

   (b) Draw a graph of a perfectly elastic supply curve.

   (c) Add a normal downward sloping demand curve to your graph. Then use the usual analysis to determine the incidence of a per-unit tax in this market. How much of the tax is paid by the buyers, and how much is paid by the sellers?

   (a) At some market price p, firms making widgets earn the market rate of return; in the long run, then, firms are indifferent between making widgets and making other things, so they are willing to produce any number of widgets at price p. At any price less than p, firms would earn less than the market rate of return; in the long run, then, no firms would be willing to produce widgets, meaning that quantity supplied would be zero at any price less than p. Similarly, at any price greater than p, firms would earn more than the market rate of return; in the long run, then, everybody would rush into the widget-making business, meaning that the quantity supplied would be infinite at any price greater than p.

   (b) A perfectly elastic supply curve for widgets is a horizontal line at some price p.

   (c) A tax on the seller would shift the supply curve up by the amount of the tax. Since the supply curve is horizontal, the equilibrium price would increase by the full amount of the tax, meaning that buyers would pay the entire tax burden. (Similarly, a tax on the buyer would shift the demand curve down, but the equilibrium price would not change, meaning that the buyers bear the full burden of the tax.) This makes sense because of the analysis above: if sellers bear part of the tax burden then they would be earning less than the market rate of return. So in the long run buyers bear the entire burden of taxes in a competitive market.

Chapter 16: Supply and Demand: Some Details

1. *Challenge* Show that it is theoretically impossible for a profit-maximizing firm to have a downward-sloping supply curve. To put this into mathematical terms, consider high and low prices (p^H and p^L, with $p^H > p^L$) and high and low quantities (q^H and q^L, with $q^H > q^L$). With a downward-sloping supply curve, q^H would be the profit-maximizing quantity at p^L and q^L would be the profit-maximizing quantity at p^H. Your job is to show that this is not possible. (*Hint:* Let $C(q)$ be the cost of producing q units of output, so that profits are $pq - C(q)$. To show that downward sloping supply curves are impossible, assume that q^H maximizes profits at market price p^L and then show that a firm facing market price p^H will make higher profits by producing q^H instead of q^L.)

   Let the relevant variables be $p^H > p^L$ and $q^H > q^L$. A downward-sloping supply curve means q^L is optimal at the higher price (so that $p^H q^L - C(q^L)$ maximizes profits at price p^H) but that q^H is optimal at the lower price (so that $p^L q^H - C(q^H)$ maximizes profits at price p^L). To proceed by contradiction, note that profit maximization at the lower market price yields
   $$p^L q^H - C(q^H) \geq p^L q^L - C(q^L).$$
   It follows that q^L is not profit-maximizing at the higher price:
   $$\begin{aligned} p^H q^H - C(q^H) &\geq (p^H - p^L) q^H + p^L q^L - C(q^L) \\ &= (p^H - p^L)(q^H - q^L) + p^H q^L - C(q^L) \\ &> p^H q^L - C(q^L). \end{aligned}$$

Chapter 17: Margins

There are no problems in this chapter.

Chapter 18: Transition: Welfare Economics

There are no problems in this chapter.