# Shipping the Good Apples Out:

# A New Perspective

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September 12, 2001

## 1 Introduction

The Alchian and Allen substitution theorem posits that a per unit tax or shipping fee applied to similar goods will increase the *relative consumption* of the higher quality good. Originally formulated in Alchian and Allen's 1964 textbook *University Economics*, the theorem is often called the "Shipping the Good Apples Out" theorem because of the empirical observation that supermarkets in apple-importing areas such as Indiana have a higher proportion of high quality apples (relative to low quality apples) than supermarkets in apple-growing areas such as Washington State. A Washington resident on holiday in Indiana might well conclude that the good apples are getting "shipped out."

Although the empirical evidence appears to support the Alchian and Allen result,[1] its theoretical foundation has been debated in a series of papers in this

[1] For example, Brown et al. (1999) find that Americans vacationing in Africa go on relatively

journal. Gould and Segall (1969) demonstrate that the result holds unequivocally only in a two-good world, and show how substitution effects can create complications in a world with a third (composite) good. Borcherding and Silberberg (1978) defend the Alchian and Allen result in the case in which the two taxed goods are close substitutes, but this special case appears to be all that can be salvaged in terms of theory. Finally, Umbeck (1980) argues for an "admission price" interpretaton of the theorem.

The main purpose of this paper is to reanalyze the Alchian and Allen result in an $n$-good world. While it is technically correct to lump all other goods into a single composite good, such a move obfuscates a valuable reinterpretation of the Alchian and Allen result in an $n$-good world. In a two-good world with only good apples and bad apples, the two kinds of apples must be substitutes for each other, and substituting out of bad apples necessarily implies substituting into good apples. In a more complicated world—say, one with good and bad pears as well as good and bad apples—the apples need not be substitutes for each other, and the decreased price of good apples relative to bad apples is not the only relevant price change. In many instances, the more important relative price changes are those of good apples relative to good pears and of bad apples relative to bad pears.

Consider, for example, the Alchian and Allen prediction that the consumption of good ($500/bottle) French wine relative to bad ($5/bottle) French wine

more high quality safaris than Europeans. They attribute this result to the additional cost of flying to Africa from America.

will be higher in the United States than in France. It is difficult to argue (à la Borcherding and Silberberg) that these goods are close substitutes; indeed, the idea that consumers substitute out of $5 wine and into $500 wine defies belief. Rather, support for the Alchian and Allen result comes from the observation that the substitutes for French wines include California wines as well as other French wines. Assume that California wines are produced at the same cost as French wines ($500 for good wine, $5 for bad) and that trans-Atlantic shipping costs are $10 per bottle. In the United States, then, one bottle of bad French wine costs three bottles of bad California wine; one bottle of good French wine costs only 1.02 bottles of good California wine. It is the substitution out of French wines and into California wines, and not the substitution out of bad wines and into good wines, that produces the Alchian and Allen result.

Properly interpreted, then, the Alchian and Allen result holds more broadly than suggested by Borcherding and Silberberg, and indeed more broadly than (though not as robustly as) originally claimed by Alchian and Allen. Hence the second purpose of this paper: to generalize the Alchian and Allen thesis and describe the conditions under which it is likely to hold.

## 2 Background

Consider a world with $n$ goods, $x_1, x_2, \ldots, x_n$, the first two of which can be thought of as, respectively, the high quality and standard quality versions of some product (e.g., good apples and bad apples). By assumption, then,

$p_1 > p_2 > 0$. Following Borcherding and Silberberg, we phrase the Alchian and Allen thesis as $\dfrac{\partial\left(\dfrac{x_1}{x_2}\right)}{\partial t} > 0$, where $x_1(p_1, p_2, \ldots, U)$ and $x_2(p_1, p_2, \ldots, U)$ are Hicksian (income-compensated) demand functions[2] and $t$ is a per unit charge applied to both goods. The chain rule gives us $\dfrac{\partial x_i}{\partial t} = \dfrac{\partial x_i}{\partial p_1} + \dfrac{\partial x_i}{\partial p_2}$, and combining this with the quotient rule we get

$$\frac{\partial\left(\dfrac{x_1}{x_2}\right)}{\partial t} = \left(\frac{1}{x_2^2}\right)\left[x_2\left(\frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial p_2}\right) - x_1\left(\frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial p_2}\right)\right].$$

Substituting in the compensated elasticities, $\varepsilon_{ij} = \dfrac{p_j}{x_i} \cdot \dfrac{\partial x_i}{\partial p_j}$, we arrive at $\dfrac{\partial\left(\dfrac{x_1}{x_2}\right)}{\partial t} = \left(\dfrac{x_1}{x_2}\right)\left(\dfrac{\varepsilon_{11}}{p_1} + \dfrac{\varepsilon_{12}}{p_2} - \dfrac{\varepsilon_{21}}{p_1} - \dfrac{\varepsilon_{22}}{p_2}\right)$. The first term here is always positive, so we will focus our attention on the second term,

$$\frac{\varepsilon_{11}}{p_1} + \frac{\varepsilon_{12}}{p_2} - \frac{\varepsilon_{21}}{p_1} - \frac{\varepsilon_{22}}{p_2}. \tag{1}$$

The Alchian and Allen claim is that (1) is positive.

# 3 A Two-Good World

With only two goods, Hicks's (1946, pages 310-311) third law

$$\sum_j \varepsilon_{ij} = 0$$

reduces to $\varepsilon_{ij} = -\varepsilon_{ii}$ and we can substitute for $\varepsilon_{11}$ and $\varepsilon_{21}$ in (1) to get

$$(\varepsilon_{12} - \varepsilon_{22})\left(\frac{1}{p_2} - \frac{1}{p_1}\right). \tag{2}$$

[2]The use of Hicksian rather than Marshallian demand curves is explained in Gould and Segall (1969).

The first term here is positive because the two goods in a two-good world must be substitutes ($\varepsilon_{12} > 0$) and own-price elasticities are negative ($\varepsilon_{22} < 0$). The second term is positive from the assumption that good apples are more expensive than bad apples ($p_1 > p_2 > 0$). We therefore get the Alchian and Allen result: $\dfrac{\partial \left(\dfrac{x_1}{x_2}\right)}{\partial t} > 0$. The intuition is that consumers are substituting out of bad apples and into good apples.

# 4 An $n$-Good World

With $n$ goods, using Hicks's third law to substitute for $\varepsilon_{11}$ and $\varepsilon_{21}$ in (1) yields

$$\frac{\varepsilon_{11}}{p_1} + \frac{\varepsilon_{12}}{p_2} - \frac{\varepsilon_{21}}{p_1} - \frac{\varepsilon_{22}}{p_2} = \frac{-\sum_{j\neq 1} \varepsilon_{1j}}{p_1} + \frac{\varepsilon_{12}}{p_2} - \frac{-\sum_{j\neq 1} \varepsilon_{2j}}{p_1} - \frac{\varepsilon_{22}}{p_2},$$

which we can rewrite as

$$(\varepsilon_{12} - \varepsilon_{22}) \left(\frac{1}{p_2} - \frac{1}{p_1}\right) + \frac{1}{p_1} \sum_{j\geq 3} (\varepsilon_{2j} - \varepsilon_{1j}) . \tag{3}$$

Equation 3 consists of two terms, which we will call (3a) and (3b), respectively. These terms identify the key factors underlying the Alchian and Allen result in an $n$-good world.

Let us begin by comparing the result in the $n$-good world (3) with that in the two-good world (2). The only mathematical difference is the addition of (3b), so ignoring this term appears to bring the $n$-good result in line with that for two goods. However, there is an important difference between (2) and (3a). In the two-good world, the two goods are forced to be substitutes, so we necessarily have $\varepsilon_{12} > 0$ and can conclude that both terms of (2) are positive. In the $n$-good

world, the two goods do not have to be substitutes, and we cannot definitively conclude that (3a) is positive.

In the $n$-good world, then, substitutability is a sufficient condition for (3a) to be positive, *but it is not a necessary condition.* The necessary condition is $\varepsilon_{12} > \varepsilon_{22}$, which simply requires that the two goods not be close complements. This suggests that the close substitutability of good apples for bad is a red herring: one might just as readily expect the consumption of French wine relative to French bricks to be higher in the United States than in France (and we will see shortly why this should be the case). It is to the development of a broader formulation of the Alchian and Allen thesis that we now turn our attention.

# 5 Sufficient Conditions (I)

Having shown that $\varepsilon_{12} > \varepsilon_{22}$ (along with our original assumption that $p_1 > p_2 > 0$) is a necessary and sufficient condition for (3a) to be positive, let us assume this condition holds and turn our attention to (3b). If the Alchian and Allen result is to hold, this term must be positive, or at least small in magnitude compared to (3a).

One approach here is to reorder goods $x_3, x_4, \ldots, x_n$ according to their substitutability with good $x_1$, defining $x^*_{1j}$ to be the $j$th closest substitute for good $x_1$ among those $n-2$ goods, and $\varepsilon^*_{1j}$ to be the associated cross-elasticity (so $\varepsilon^*_{i1} \geq \varepsilon^*_{i2} \geq \ldots \varepsilon^*_{i,n-2}$). Making similar definitions for $x^*_{2j}$ and $\varepsilon^*_{2j}$, we can rewrite

(3b) as

$$\frac{1}{p_1}\sum_{j=1}^{n-2}\left(\varepsilon^*_{2j}-\varepsilon^*_{1j}\right). \tag{4}$$

For this summation to be nonnegative, it is sufficient for each term to be nonnegative, i.e., for the $j$th closest substitute for good $x_2$ to be a better substitute (or poorer complement) than the $j$th closest substitute for good $x_1$. In other words, it is sufficient for good $x_2$ to have better (or equivalent) substitutes and poorer (or equivalent) complements than good $x_1$.

The sufficient conditions that come from this analysis are as follows: *If two goods are not close complements* ($\varepsilon_{12} > \varepsilon_{22}$), *and if the lower priced good has better (or equivalent) substitutes and poorer (or equivalent) complements than the higher priced good* ((4) $\geq 0$), *then the imposition of a per unit charge will increase the relative consumption of the higher priced good, i.e., the higher priced good will be "shipped out."*

It is this set of sufficient conditions which has been the focus of previous analyses. Gould and Segall (1969) highlight the importance of the substitutability requirement: if better substitutes exist for the higher priced good, it may be the lower priced good that gets shipped out. Mathematically, (4) can be negative, leaving the overall sign of (3) in doubt.

Borcherding and Silberberg (1978) respond that two goods that are close substitutes for each other must necessarily be fairly evenly matched in terms of substitutes and complements, and hence that the Alchian and Allen thesis must hold for these goods. Mathematically, for two goods that are very close substitutes, the terms of (4) must all be close to zero, making (3a) (which is

necessarily positive) the dominant term in (3).[3]

Of course, the substitutability requirement can be met even if the two goods are not substitutes for each other, and in this case the Alchian and Allen result should still hold. But theory gives us no basis for such claims in any individual instance. Why should we believe that French bricks have better substitutes (and poorer complements) than French wines?

## 6 Sufficient Conditions (II)

Returning to (3), we can see an alternative (and unexplored) set of sufficient conditions that arises if $p_1$ is large relative to $p_2$. This price difference swamps all but the most powerful Gould and Segall-type effects, as we can see by comparing (3a) and (3b) when $p_1 \gg p_2$. Barring exceptional circumstances, then, (3) will be dominated by $(\varepsilon_{12} - \varepsilon_{22})\left(\frac{1}{p_2}\right)$.[4] So an alternative set of sufficient conditions is this: *If two goods are not close complements* ($\varepsilon_{12} > \varepsilon_{22}$) *and are not close in price* ($p_1 \gg p_2$)*, and if the higher priced good does not have much better substitutes (or much poorer complements) than the lower priced good, then the imposition of a per unit charge will increase the relative consumption of the higher priced good, i.e., the higher priced good will be "shipped out."*

It is this formulation of the Alchian and Allen thesis that yields a prediction about French bricks and French wine. If the trans-Atlantic shipping charge is

[3] Intuitively, we have essentially returned to a two-good world: other goods don't matter, and consumers are substituting out of bad apples and into good apples.

[4] Note that this also follows directly from (1).

based on weight, we have good reason to expect the consumption of $200/lb French wine relative to $.10/lb French bricks to be higher in the United States than in France.

The intuition here is the same as that in the example of cheap and expensive French wines in the Introduction. American consumers are *not* substituting out of cheap French wine and into expensive French wine, much less out of French bricks and into French wine. Rather, they are substituting out of French products and into American products, and the relative price changes that lead to those substitutions are more pronounced for cheaper French products. [5]

This formulation of the Alchian and Allen thesis allows for its application in new circumstances, and the intuition behind it provides a firmer basis for many "shipping out" analyses, such as the import tariffs considered in Falvey (1979). A luxury tariff of $100 per item levied on imported Italian fashions is likely to increase purchases of $500 Gucci handbags relative to $100 Armani handbags—even if Armani afficionados wouldn't be caught dead in Gucci.

# 7 Conclusion

The Alchian and Allen thesis must be applied carefully, as we can see by considering units. None of the analysis above appears to take units into account, and without specifying the units of measurement for $x_1$ and $x_2$ it does not make

[5] To see this result mathematically, consider some good $x$, the unit price of which is $q$ units of some other good $y$ (so that $p_x = qp_y$). If a tax $t$ is applied to good $x$, its unit price rises to $q\left(1 + \frac{t}{p_x}\right)$ units of $y$. The magnitude of this relative price change varies inversely with $p_x$.

much sense to label one the more expensive good: one case of "cheap" wine might well be more expensive than one glass of "expensive" wine.

The resolution of this issue lies in the observation that the "shipping the good apples out" phenomenon occurs when an *equivalent* charge $t$ is applied to two goods. Consequently, the nature of the charge determines the correct units for the analysis. For a tax levied per bottle, goods $x_1$ and $x_2$ must be measured in bottles (or some common multiple, such as cases). For a per pound transportation charge, goods $x_1$ and $x_2$ must be measured by weight. And for a per dollar charge (e.g., an ad valorem tax), goods $x_1$ and $x_2$ must be measured in dollars, or some other monetary unit. This final observation provides some intuition for the result (suggested in Barzel, 1976, and made explicit in Falvey, 1979) that we should *not* anticipate the shipping out of good apples under an ad valorem tax: in this case the correct measurement unit is dollars, so $p_1 = p_2$ and it becomes impossible to even classify one of the goods as being of high quality, much less predict that it gets shipped out.[6]

Carefully defining goods $x_1$ and $x_2$ also helps in analyzing situations in which the *customer* is transported to the *good* rather than the good to the customer. Gould and Segall (1969) introduce examples of this sort, using them to cast doubt on the hypothesis that the good apples get shipped out: "How often is it heard, for example, that the way to get really good farm produce is to drive out to the country and buy it at a roadside stand or that one must go to Maine to get truly delectable lobsters?" Borcherding and Silberberg (1978)

[6]Mathematically, (3a) is zero, leaving the overall sign of (3) appropriately indeterminate.

respond that such examples in fact *support* the Alchian and Allen result: "[I]t does not matter if the goods are shipped to the consumers or the consumers are shipped to the goods. Going to Maine... involves a transport cost to people not from Maine... What the [Alchian and Allen] proposition predicts, therefore, is that tourists in Maine will consume, on average, higher quality lobsters than natives..."

Once again, we must seek clarity about the goods in question. Since $x_1$ and $x_2$ are good and bad lobsters in the case in which the *lobsters* are getting shipped out, it is natural to consider this possibility when it is the *tourists* that are getting shipped out. But there is a problem with this formulation: the transportation charge is not a per unit charge on lobsters—once in Maine, a tourist can purchase as many lobsters as she wishes for the same price that locals pay. So the Alchian and Allen thesis does not apply.[7] An alternative view is that the goods are *trips to Maine*, with $x_1$ being a week-long trip to "live the good life" (and eat good lobster) in Maine and $x_2$ being a week-long trip to "live the mediocre life" (and eat mediocre lobster) in Maine. An increase in travel costs is therefore a per unit charge added to both goods, and we can apply the Alchian and Allen thesis.

In doing so, however, we should be aware that "good living in Maine" and "mediocre living in Maine" are unlikely to be close substitutes for each other.

[7] An additional problem is that tourists may face time constraints that restrict their possible consumption bundles: the ability to buy 3 bad lobster dinners for the price of 1 good lobster dinner is of limited interest to a tourist scheduled to leave Maine the next morning. This problem invalidates Umbeck's (1980) "admission price" argument.

A more compelling story is that their close substitutes are (respectively) good and mediocre living at home (or some other vacation spot). When travel costs to Maine rise, some potential visitors substitute out of good living in Maine and into good living at home, and others substitute out of mediocre living in Maine and into mediocre living at home. This substitution effect is stronger for the latter group, as can be seen by imagining that a week of good living anywhere in the U.S. costs $400 and that a week of mediocre living costs $200. If transportation costs to Maine increase from $100 to $200, a week of mediocre living in Maine increases in cost by 33%, from 1.5 weeks of mediocre living at home to 2 weeks; a week of good living in Maine increases in cost by only 20%, from 1.25 weeks of good living at home to 1.5 weeks. This differential impact leads to the Alchian and Allen prediction: the "good living" tourists get shipped out—and therefore we can expect the average tourist in Maine to consume higher quality lobster than the average native.

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